3D slope stability analysis by the using limit equilibrium method analysis of a mine waste dump

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ABSTRACT: The authors dealt with a difficult project in northern Peru involving a three-dimensional (3D) slope stability analysis of a mine waste dump. During the extensive geotechnical investigation program carried out for this project, it was determined that the waste dump foundation was composed by veryof heterogeneous soil layers, most of them compromising the stability of this facility. As consequence, , so that there could be sections with high factors of safety (FS) and others nearby with poor stability conditions were encountered. In order to overcome such apparent instability, commercially-available software was used to create a 3D model that included 11 different soil layers and the complicated mine waste layout. The results showed a 14% to 18% FS increase compared with the minimum two-dimensional (2D) FS. M, mostly this difference was because the 3D failure surface did not only cross weak soil layers, but also strong ones, neglecting the over-conservative simplification assumed with the 2D sections for this particular analysis. A greater difference between 2D and 3D analysis is expected for more complex cases.

1 INTRODUCTION

Modern mining operations manage many structures specifically designed for storing materials with high engineering and environmental standards. These materials are removed, produced or used by the mining operation itself; some examples of these materials are: tailings, ore, mine waste or low grade ore, water, topsoil, etc. Mine waste is removed from the open pit or underground mining operations in order to expose and mine higher grade materials. In the last years, mine waste is produced and disposed in higher quantities due to the increasing mineral demand and the diminishing high grade ore reserves.

In Peru, mining operations deal with the rugged, mountainous and many times aggressive Andes' terrain. For this reason, when designing a mine waste dump, problems such as slope stability and locating a suitable site for disposing the mine waste are found, among other issuesencountered. Common mine waste dump layouts include different combinations of valley-fill, side-fill, heaped and ridged configurations. Furthermore, moraine, residuals and alluvial soil deposits sometimes in loose conditions, are usually found together in "suitable" locations for mine waste dumps.

The geotechnical design of mine waste dumps involves site investigations that include geotechnical-geological mapping, boreholes, test pits, geophysical tests and *in situ* tests depending on the type of foundation soils, and a laboratory program to properly characterize the geotechnical properties of mine waste and foundation soils. Current state of practice of slope stability analysis involves the use of the limit equilibrium (LE) method in 2D sections that are chosen to be representative of the structure to be analyzed. However, most foundation soils in the Peruvian

Andes are heterogeneous and, in combination with the complicated mine waste layouts, it is commonly difficult to select appropriate cross-sections to analyze its stability, so the tendency is to be conservative.

This paper focuses on the slope stability analysis of a mine waste dump located in northern Peru, that involved many and heterogeneous soils layers in its foundation and a complicated mine waste layout. A 2D slope stability analysis was over-conservative and insufficient, and in consequence a large and realistic 3D model was built and analyzed in order to overcome such apparent instability found in the 2D analysis. This 3D analysis represents a benchmark for Peruvian geotechnical practitioners and the mining industry, and the starting point in performing 3D slope stability analysis in mining facilities.

To maintain consistency with the terminology used by previous researchers and for a better understanding, a clear distinction is made with the terms "method" and "procedure". LE is a method whereas Bishop's simplified (Bishop 1955), Janbu's simplified (Janbu et al. 1956, Janbu 1973), Spencer's (Spencer 1967) and Morgenstern & Price's (Morgenstern & Price 1965), are procedures within the LE method.

2 THE LIMIT EQUILIBRIUM METHOD

In current practice, slope stability analyses are usually solved with the LE method using 2D procedures, due to their simplicity and general understanding. Calculations usually consist of computing a factor of safety (FS) using one of several LE procedures, each one of them using the same definition of the FS and compute the FS using equations of static equilibrium (Duncan & Wright 2005). In LE, the FS is defined as the minimum factor by which the soil strength must be divided to bring the slide mass to the verge of failure. The soil mass is assumed to be at the verge of sliding failure and the equilibrium equations are solved for the unknown FS (Akthar 2011). The FS equations of static equilibrium for all LE procedures can be written in the same form if it is recognized whether moment and/or force equilibrium is explicitly satisfied (Fredlund & Krahn 1977).

2D procedures assume that the slope is infinitely wide (plane strain) in the direction perpendicular to the plane of interest and therefore, 3D effects are negligible. Clearly, all slopes and failure surfaces are not such infinitely wide and generally not symmetric, so 3D effects influence the stability of the slope. Gitirana et al. (2008) found that analyzing 3D models can lead to differences in the lowest FS between 15% and 50%.

2.1 *Two-dimensional limit equilibrium procedures*

Traditionally, 2D LE the slice procedure is are used for solving 2D LE stability analysis for classified into circular, non-circular and slice procedures, the last ones being the most used in practicefailure surfaces. Fredlund & Krahn (1977) compared the most commonly used 2D procedures of slices in terms of consistent procedures for deriving FS equations and solved these equations for a case of composite failure surface, partial submergence and line and earthquake loading.

Table 1 shows the principal characteristics of the most commonly used 2D LE procedures. Bishop's Simplified and Janbu's Simplified procedures are two of the non-rigorous procedures often used by geotechnical practitioners for comparison purposes as they yield acceptable values of FS compared to rigorous procedures. Spencer's, Morgenstern & Price's and the General Limit Equilibrium (Fredlund et al. 1981) are rigorous procedures that have been successfully implemented in most of the commercially-available software and are among the most used procedures.

2.2 *Three-dimensional limit equilibrium procedures*

Duncan (1996), Akhtar (2011) and Kalatehjari & Ali (2013) made extensive reviews of existing 3D LE procedures available since 1969. However, after more than four decades, geotechnical practitioners have not yet accepted 3D procedures compared with the several 2D procedures used nowadays. Extending 2D LE procedures to 3D necessitates more assumptions for rendering the problem statically determinate (Akthar 2011) and building the 3D geometry of a real slope and determining its critical failure surface are issues still not well understood for practitioners.

Table 2 shows the most important characteristics of the procedures reviewed by Akhtar (2011) and Kalatehjari & Ali (2013). As it can be noted, most of the 3D procedures are direct extensions of 2D procedures of slices by adding the third dimension and changing them into columns.

Table 1. Main characteristics of commonly used two-dimensional limit equilibrium method procedures (modified after Duncan & Wright 2005).

Procedure	Equilibrium condition satisfied	Assumptions
Bishop simplified	Vertical and overall moment	Side forces are horizontal
Janbu simplified	Vertical and horizontal	Side forces are horizontal
Spencer	All conditions	Interslice forces are parallel and normal force acts at the center of the base of the slice
Morgenstern & Price's	All conditions	Interslice shear force is related to the interslice normal force by $X/E = \lambda f(x)$
General Limit Equilibrium	All conditions	Interslice shear force is related to the interslice normal force by $X/E = \lambda f(x)$

Akthar (2011) concluded that 3D FS is greater than 2D FS and showed there were serious inaccuracies involved in the studies that indicated the opposite. For example, many of the 3D procedures are based on the ordinary method of slices (Fellenius 1927), which have proven to calculate erroneous FS by as much as 50% (Duncan & Wright 1980). Additionally, past researchers used assumptions and geometries that do not represent field behavior. Furthermore, less than a half of the references utilize field case histories to validate their formulations, and in these only field case histories that were applicable to their formulations were studied instead of a wide range of case histories with different conditions.

Despite the disagreement between researchers and the general confusion between practitioners, commercially available software usually perform 3D analysis using columns and published 3D extensions of 2D LE procedures. 3D software must properly model slope geometry, material properties and general failure surfaces.

2.3 *Limit equilibrium accuracy*

Duncan (1996) concludes that even though it is difficult to know the correct FS, it is possible to determine sufficiently accurate values of FS. This conclusion is based on findings that all methods that are considered to be accurate provide a similar FS. The continuum mechanics method has the ability to model complex problems without simplifying assumptions, which is an big advantage over the LE method. Within the continuum mechanics method, finite element (FE) and finite difference (FD) are different procedures. Researchers such as Griffiths & Lane (1999), Chugh (2003) and Griffiths & Marquez (2007) have performed slope stability analyses using FE and FD procedures and show that these procedures provide comparable results to LE procedures.

Akhtar (2011) compared LE, FE and FD procedures analyzing 2D and 3D slope problems. For 2D LE analyses, the procedures of Morgenstern & Price and Spencer yielded reasonable estimates of the 2D FS for any shape of failure surface;, however, Spencer is preferred because of Morgenstern & Price needs to select an appropriate interslice force function. Bishop and Janbu simplified procedures are also suitable for routine analyses.

For 3D LE analyses, accepted 3D extensions of Bishop's, Morgenstern & Price's and Spencer's 2D procedures provided comparable results with continuum methods and are within 3% of each other. As Morgenstern & Price's and Spencer's procedures satisfy all conditions of equilibrium, they are preferred over Bishop. However, as in 2D, Morgenstern & Price's procedure requires the selection of an appropriate function for inteslice forces, so the 3D extension of Spencer's procedure is preferred. Akhtar (2011) also suggests correction factors for the 3D extension of Janbu's procedure. Using his factor, Bishop's and Janbu's procedures are viable alternatives to the 3D extension of Spencer's procedure.

Table 2. Principal characteristics of existing 3D procedures based on the limit equilibrium method (modified after Akhtar 2011 and Kalatehjari & Ali 2013).

Procedure	Theoretical basis	Slip surface	Factor of safety ratio found (3D/2D)
Anagnosti (1969)	Morgenstern & Price (1965)	Unspecified	>1
Baligh & Azzouz (1975)	Swedish circle (Fellenius 1922)	Cylindrical with conical or ellipsoidal ends	>1
Hovland (1977)	Ordinary method of slices (Fellenius 1927)	Cylindrical with conical ends	>1 for cohesive soils, <1 for cohesionless soils
Chen & Chameau (1983a, b)	Spencer (1967)	Cylindrical with conical or ellipsoidal ends	>1 for cohesive soils, <1 for cohesionless soils
Thomas & Lovell (1988)	Spencer (1967)	Symmetrical	>1 for cohesive and not always for cohesionless soils
Dennhardt & Forster (1985)	Limit equilibrium method	Ellipsoidal	>1
Leshchinsky et al. (1985)	Limit equilibrium method and variational calculus	Spherical and cylindrical	>1
Ugai (1985)	Limit equilibrium method and variational calculus	Cylindrical with curved ends	>1
Hungr (1987)	Bishop (1955)	Rotational with circular central section	>1
Ugai (1988)	Fellenius (1936), Bishop (1955) and	Cylindrical with Spencer (1967)	>1 for cohesive soils, ellipsoidal ends <1 for cohesionless soils
Gens et al. (1988)	Swedish circle (Fellenius 1922)	Cylindrical with planar or curved ends	>1
Xing (1988)	Ordinary method of slices (Fellenius 1927)	Symmetrical elliptic with circular vertical cut	>1
Hungr et al. (1989)	Bishop (1955) and Janbu's Simplified (Janbu et al. 1956, Janbu 1973)	Symmetrical and rotational	>1
Leshchinsky & Huang (1992a, b)	Limit equilibrium method and variational calculus	Expansion of a log-spiral function	>1
Lam & Fredlund (1993)	General equilibrium method (Fredlund et al. 1981)	Generalized rotational	>1
Huang & Tsai (2000)	Bishop (1955)	Asymmetrical	>1
Hungr (2001)	Morgenstern & Price (1965)	Symmetrical	>1
Chang (2002)	Sliding block analysis	Asymmetrical	>1
Huang et al. (2002)	Janbu's generalized (Janbu 1954, Janbu 1973)	Generalized	>1
Chen et al. (2003)	Spencer (1967)	Generalized rotational	>1
Jiang & Yamagami (2004)	Spencer (1967) and variational calculus	Symmetrical rotational	>1
Cheng and Yip (2007)	Bishop (1955), Janbu's simplified (Janbu et al. 1956, Janbu 1973) and Morgenstern & Price (1965)	Spherical	>1
Zheng (2009)	Limit equilibrium method	Generalized	>1
Sun et al. (2012)	Morgenstern & Price (1965)	Generalized	>1

3 SLOPE STABILITY ANALYSIS IN CURRENT PRACTICE

Most slope stability analyses in current practice are solved using 2D LE procedures, such as Morgenstern & Price, General Limit Equilibrium and Spencer. Spencer procedure is preferred over the other ones because of their need for selecting appropriate functions for interslice forces and because they present convergence problems.

In 2D analyses, a number of 2D cross-sections are selected in such waya manner, that they represent the inherent 3D problem, simplifying 3D geometry, 3D failure surface shape, and length and soil variability. Hence, the accuracy of the analysis depends on the ability to select cross-sections, determine the direction of sliding (DOS) and even assume over-conservative simplifications.

Figure 1 shows some slope stability problems the authors have dealt with in the mining industry. In each one of them, cross-sections used to determine the 2D FS are shown in plan view. For these cases, 2D analyses are considered appropriates because of the materials properties and configuration involved in the analysis do not vary significantly in the perpendicular direction of the cross-section (and the assumed DOS).

Figure 2 shows examples where 2D analyses may yield over-conservative FS. Cross-sections used to analyze each example are also shown in plan view. It can be noted that a large number of sections are used, some of them specifically modeled for analyzing particular soil layers and geometry configurations; it is hard to determine the main DOS. It is important to mention that in all of these examples, a extensive program of geotechnical investigations were carried out to characterize all materials involved in each section.

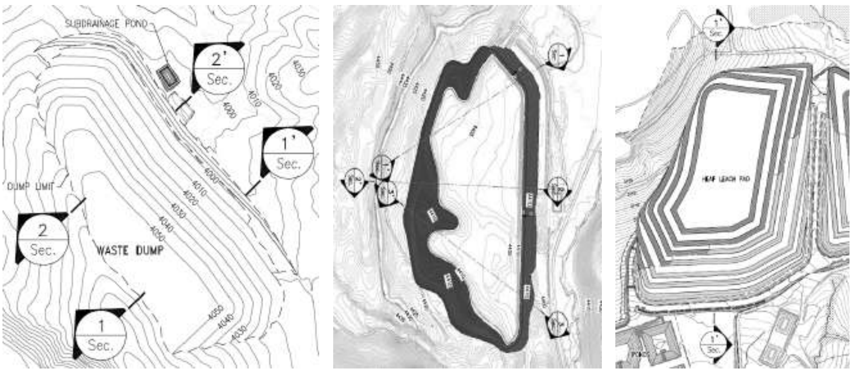


Figure 1. Mining facilities such as waste dump (left), tailings dams (middle) and heap leach pad (right) where 2D limit equilibrium slope stability analyses are sufficient.

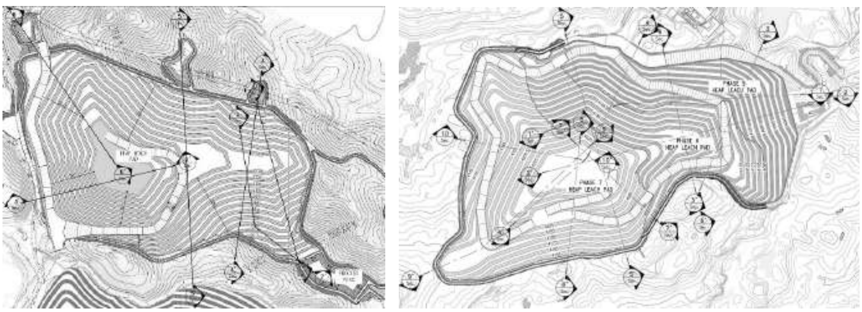


Figure 2. Examples of heap leach pads where 2D LE will yield to over-conservative results.

The focus of this paper is to present a 3D analysis of a mining facility, such as the ones showed in Figure 2. During the authors' consulting experience, they have designed and analyzed many mining facilities, such as tailing dams, heap leach pads, mine waste dumps, earth dams, among others, all of them located in the rugged, mountainous and many times aggressive South American Andes' terrain. Dykes and dams are usually located in very narrows valleys, and heap leach pads and mine waste dumps are located in heterogeneous terrains and in consequence they have complicated layouts. Furthermore, many times these facilities are built over loose moraine, residual and alluvial soil deposits. Stabilization solutions based on 2D analysis for cases such as the ones showed in Figure 2, include the use of stability platforms, gentle slopes, buttressing, massive inadequate material removal, and also limiting the height of the facility. To overcome such apparent 2D instability and over-conservative design (and high associated costs), a 3D model should be created.

As the accuracy of the LE method have been cleared out in previous sections, this method is recommended for 3D slope stability analysis mainly because its mechanics and parameters are easier to understand. In consequence, model geometry, failure surface and DOS are the main issues when a 3D model is analyzed. Model geometry is usually build based on interpolating input cross-sections or creating several 3D surfaces, so realistic models depend on the amount of topographical, geological and geotechnical information available. Failure surfaces should match field failure mechanisms.

Akhtar (2011) reviewed field case histories and determined the nature of slides. Rotational failures are ellipsoidal, with aspects ratios varying from 0.8 to 2.67. Translational failures commonly occur when a stronger material underlays a weaker one; Akthtar (2011) presented a method that includes the effect of side resistance based on the research of Arellano & Stark (2000). A 3D analysis is strongly recommended for back-analysis of slope failures, because of a 2D analysis back-calculates in a conservative way the shear strength (Arellano & Stark 2000). The unique DOS is hard to determine because of common commercially available software analyzes models in one direction only. It is strongly recommended to put a lot thought inconsider choosing the most suitable software, depending on the characteristics of the slope stability analysis.

4 THREE-DIMENSIONAL SLOPE STABILITY ANALYSIS OF A MINE WASTE DUMP

4.1 *General background*

One of the fastest-growing gold producer mining companiesy in Peru had a long-term expansion plan that included the expansion of its heap leach pad, design of a brand new tailing dam, and the redesign of its mine waste dump. The most challenging one the authors dealt with was the last one, not only because the limited space available, but also because of previous geotechnical field work had shown soft alluvial clayey soils and loose residual soils around the existing dump. In addition, the redesign made the mine waste layout complicated because of the topography. Figure 3 shows the original design and the proposed layout.

The mine waste dump needed a redesign to get reach a capacity of 113 million tones (66 million cubic meters), within an area of 118 Ha. The angle of repose of the waste rock formed a slope of 1.5H:1V and had an overall slope of 2.5H:1V.

4.2 *Geotechnical characterization*

The authors' consulting company worked on the redesign of the mine waste dump. This project included and extensive geotechnical investigation program for waste rock and soil foundation characterization. This investigation involved a large quantity of field work which included; geological-geotechnical mapping, 123 test pits, 31 dynamic penetration tests, 91 standard penetration tests, 30 large penetration tests and 21 geotechnical boreholes, among an extensive laboratory test program in order to model accurately the soil and rock materials founded. Shallow water table was conservatively assumed at constant level.

Site investigation and laboratory tests found many soil layers in the foundation, each one with different depth, location and shear strength parameters. Figure 4 shows two cross-sections (previously presented in plan view in Figure 3) and; soil layers identification and strength parameters are presented in Table 3.

Figure 3. Original design (left) and proposed layout (right) of the mine waste dump.

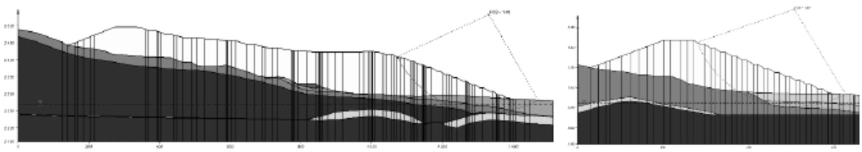


Figure 4. Critical cross-sections 1-1’ (left) and 2-2’ (right) from Figure 3 for Zones 1 and 2, respectively.

Table 3. Mohr-Coulomb shear strength parameters for soil layers involved in the analysis.

Material	Specific Total unit weight	Cohesion	Friction angle
	(kN/m^3)	(kPa)	(º)
Mine waste rock	19	0	37
Soft clay	14	0	12
Coluvial soil	17	0	30
Clayey alluvial soil	19	30	16
Silty alluvial soil	18	10	13
Sandy alluvial soil	18	14	23
Gravelly alluvial soil	19	0	30
Sandstone residual soil	19	25	18
Granodiorite residual soil	17	30	17
Bedrock	23	120	20

4.3 *Two- dimensional slope stability analysis*

Given the heterogeneous soil layer distribution and waste mine layout, the dump was divided in two areas named Zone 1 and Zone 2, (as shown in Figure 3), each one having different soil layers distribution in their foundations and also waste rock layout. The most critical sections (showned in Figure 3 and 4) of each zone were used in a short term 2D LE slope stability analysis (using Spencer’s procedure) as a first order conservative approach. Results showed FS lower than the minimums required by the design criteria for static and pseudo-static conditions on both sections. Additional cross-sections were evaluated to further analyze the effect of foundation configuration for each zone; results showed lower and higher FS around the main critical sections. Table 4 presents FS calculated for the two most critical sections from the 2D analysis. No stability platforms, pre-loads, buttressing or change in the overall slope were considered due to the limited space and minimum storage capacity specified. Thus,, so a 3D analysis was considered to overcome such apparent instability.

4.4 *Three-dimensional slope stability analysis*

As previously pointed out, a realistic and comprehensive 3D model should be built that accounts for sufficienttaking into account the enough information to properly represent the geometry of the slope, soil layer distribution and geotechnical parameters. For this particular case, detailed topography and waste rock layout were available as well as enough boreholes, laboratory and *in situ* test for modeling the foundation. In consequence, a 3D analysis would improve the over-conservative simplifications assumed in the previous 2D analyses.

The SVSLOPE-3D software package (Fredlund & Thode 2011) from SoilVision Systems (2011) was used to create a 3D model for each zone. SVSLOPE-3D performs LE analyses using the columns method and 3D accepted extensions of classical 2D procedures (Fredlund & Thode 2011), accordingly to most of the recommendations presented on Akhtar's (2011) research.

SVSLOPE-3D allows creating a 3D model by, among other methods, interpolating cross-sections, which was the best option for this analysis because of the numerous soil layers for each zone and the abundant borehole data. In order to create a 3D model for two sections, fifteen geotechnical sections were developed for Zone 1 and nineteen for Zone 2 by geotechnical and geologicalst engineers. Cross sections were separated laterally, separated each one by 50 m. Each section was drawn in a CAD software and all the point data was exported to Microsoft Excel files. Subsequently, SVSLOPE-3D imported every Excel file and successfully interpolated over 30 sections in both models. During this process 11 layers were created, representing accurately the mine waste dump's complex foundation and layout, even modeling small lenses of loose soils. Figure 5 shows models for Zone 1 and Zone 2 as presented in SVSLOPE-3D. Figure 6 presents sections for each zone, where variability can be noted.

The DOS was assumed to be parallel to the slope direction of both the layout and topography. SVSLOPE-3D allows searching for the DOS;, however, sections were built parallel to the assumed DOS in order to focus the analysis on 3D failure surface shape and location. The failure surface search method used was the 3D grid and tangent, where several searches were performed, each one increasing the grid density and location in the most critical zones. As a rotational failure was expected, the failure surface shape was assumed to be ellipsoidal, and in the analysis the critical aspect (eccentricity) ratio was determined by trial and error. The 3D extension of Spencer's procedure was used in order to compare it with the 2D results. Finally, soil parameters were the same from the 2D analysis since previous effort was made in characterizing soil variability in both zones.

Table 4 presents the results from the 3D analyses for both zones. Figure 5 shows 3D critical failure surfaces for Zone 1 and Zone 2 static analysis. A ratio of 3D/2D FS is presented, comparing the minimum 3D FS and the minimum 2D FS for the models (Cavounidis 1987), showing that 3D FS is about 14% to 18% greater than 2D FS. Table 4 also presents the critical aspect ratios determined for both Zones. It can be noted that 3D FS is greater than 2D FS for static and pseudo-static conditions and for both zones. It is important to mention as well that the location of the 2D critical section does not necessarily match the location of the center of the 3D failure surface.

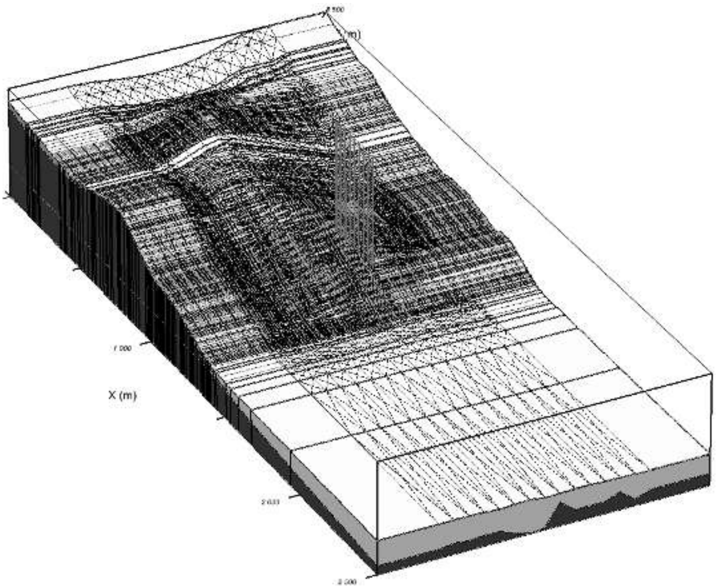
Table 4. Calculated factors of safety from 2D and 3D analysis for the mine waste dump.

Analysis type	Section/Zone analyzed	Static factor of safety	Pseudo-static factor of safety	Static 3D/2D ratio	Pseudo-static 3D/2D ratio	Aspect ratio
2D	1-1'	1.430	1.093	-	-	-
3D	Zone 1	1.689	1.255	1.181	1.148	1.0
2D	2-2'	1.237	0.932	-	-	-
3D	Zone 2	1.419	1.066	1.147	1.144	1.7

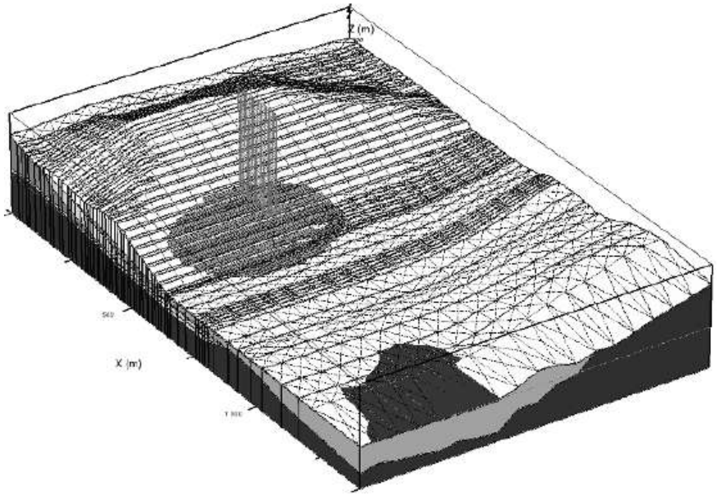
As 3D FS was greater than the minimum required by the design criteria and the 2D analysis was shown to be over-conservative; the stability of the mine waste dump was guaranteed by the authors. Nevertheless, the "3D effect" was expected to be higher, but the irregular layout and heterogeneous foundation had different effects on the overall FS for Zones 1 and 2, respectively. Figure 7 shows which soil layer parameters are used to calculate the shear resisting force in the base of each column for the static analysis. It can be seen that the failure surfaces cross weak and strong soil layers in the perpendicular direction of the slope movement.

Because of the potential variability of the operational aspects related to the waste rock quality and geotechnical properties and rate of raise of the waste dump, geotechnical instrumentation for monitoring the dump behavior was implemented. Also a constant reevaluation of the waste rock shear strength parameters were recommended.

This analysis represents a benchmark for Peruvian geotechnical practitioners and the mining industry, and is a starting point for performing 3D stability analysis in mining facilities.



(a)



(b)

Figure 5. 3D models for Zones 1 (lefta) and 2 (rightb). Critical failure surfaces for the static analysis are shown for both models.

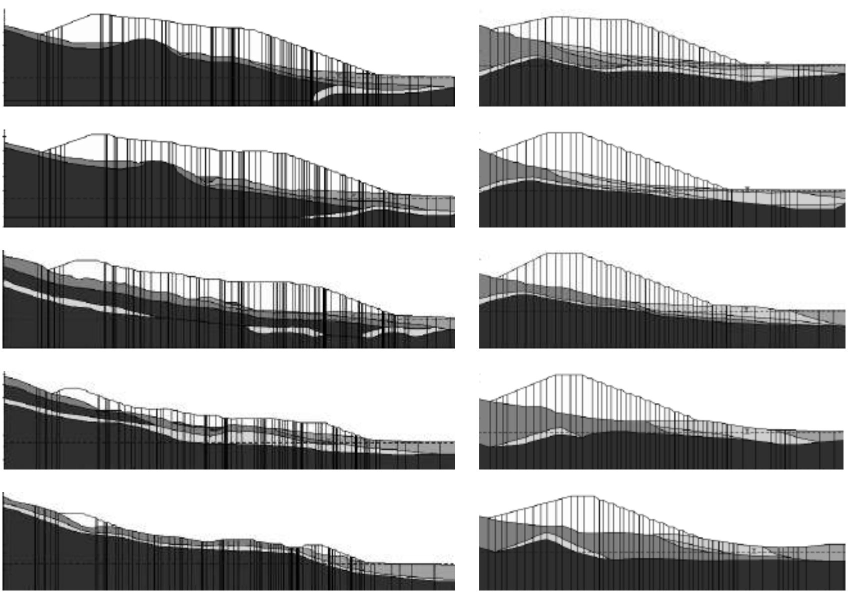


Figure 6. Sections from models of Zone 1 (left) and 2 (right). Note mine waste rock layout and soil layer variability.

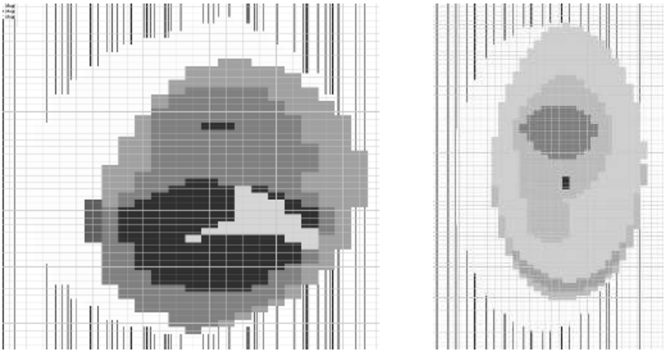


Figure 7. Caption showing which soil layers contribute to shear resisting force calculation in the base of each column for the failure surfaces in Zone 1 (left) and 2 (right). Direction of sliding from left to right.

5 CONCLUSIONS AND RECOMMENDATIONS

Current state of the art for evaluating slope stability involves 2D LE analysis and the use of rigorous procedures, such as Spencer, in cross-sections which that neglects 3D effects. 3D analysis can be a viable solution when geometry and soil strength parameters are know in detail and it is relatively simple to assume the DOS and the failure surface shape. Using 3D analysis in design needs special considerations and understanding on the mechanics of 3D modeling and should be performed with care. It is the authors opinion that 3D analysis can be used in design if 2D analysis are over-conservative and its simplifications are too unrealistic.

Software's The potential for 3D analysis in a given slope stability software should be evaluated before its use, as it needs to accurately model slope geometry and soil strength. Additionally, it must allow the user to evaluate the methods and techniques of 3D LE procedures, understand failure surfaces search techniques and failure surface geometry.

A 3D analysis of a mine waste dump was carried out. The 2D analysis of the dump was over-conservative because it had a complex mine waste rock layout and was located in a site with complicated topography and foundation. A complete and detailed geometry of the slope and enough geotechnical data to model all the materials involved, were available to built 2 create two large 3D models. 3D FS proved to be greater than 2D FS in a real and complete 3D analysis, and these results were used in design. However, common design of slopes should be performed using 2D analysis to maintain the current conservatism inherent in a 2D analysis. Regulatory agencies and codes should specify a "minimum 2D FS" as a design criteria and further research is needed to properly specify proper criteria for 3D analyses.

6 ACKNOWLEDGMENTS

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