

3D dynamic analysis of a valley–fill heap leach pad

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Abstract

In countries such as Peru, located in complex and active seismic regions, it is vital to consider the seismic demand on site during design of earth structures. In the mining environment, heap leach pads and their liner system are considered more sensitive to seismic induced displacements than other mine facilities due to the potential for geomembrane tearing during seismic events, which can lead to environmental damage and economical detriment. Additionally, heap leach pads in these regions, and particularly in Peru, are usually built within narrow valleys where the three-dimensional nature of these locations may considerable influence their seismic behavior. This kind of complexity requires an adequate characterization and definition of the shear strength and deformational properties of the materials involved in its design.

This paper presents a Peruvian case study of a valley-fill heap leach pad where the design was defined by its seismic behavior, and a three-dimensional (3D) dynamic analysis was performed using the software *FLAC*^{3D} (Itasca, 2012) in order to validate 2D seismic analysis for the heap.

A large set of geotechnical information was used for the analysis which included state-of-the-art characterization of static and dynamic properties of leached ore and interface (liner system). The shear strength and deformability properties of the leached ore were defined considering a homogeneous media and implementing a non-linear variation of these with the confining stresses. The shear strength of the interface was determined using large scale direct shear tests. The dynamic properties of the leached ore

were studied using geophysical surveys directly on the heap and using combined resonant column and torsional shear tests as well as cyclic triaxial test. The approach of Yegian et al. (1998) was used to define the soil-geomembrane interface dynamic properties. The results allowed to authors both to validate 2D simplified and dynamic analysis as well as to understand the behavior of the ore and interface in the valley during a seismic event.

Introduction

The Peruvian mining industry operates at high altitude in the Andes, where its topography is very aggressive and unfavorable for heap leach pad (HLP) design and construction. A standard project can operate at altitudes higher than 3,000 meters above sea level, where the only place available for earth mining structures is usually narrow valleys. The design of earthworks, liner systems, solution collection systems and first lift stacking usually involve special and specific design criteria that differs significantly from the ones used in conventional HLP constructed in almost ideal conditions, such as flat terrains at much lower altitudes.

In countries such as Peru and Chile, which are subjected to strong seismic events, seismic stability analysis of HLP is paramount during design stages and is regularly performed through pseudo-static analysis and less often by the calculation of seismic-induced permanent displacements (SIPD). The approach for SIPD calculation varies from simplified methodologies to fully coupled dynamic analysis (Reyes and Pérez, 2015) and is focused on determining the magnitude of displacements induced by seismic forces in the soil-geomembrane interface of the HLP liner system. The analysis methodologies are whether from one-dimensional (1D) or two-dimensional (2D) nature; however, no previous study has assessed the influence of the three-dimensional nature of valleys for HLP on both the heap and interface dynamic response.

Based on geotechnical site investigations, advance laboratory testing and previous studies related to seismic analyses of HLP, this paper presents the 3D dynamic analysis of a valley-fill heap leach pad located in northern Peru. This evaluation was part of a large set of seismic analyses that included 1D seismic response analysis, simplified procedures for the calculation of SIPD and 2D and 3D fully coupled dynamic analyses. Parra et al. (2016) and Regalado et al. (2016) discuss in detail the dynamic properties of the leached ore and the overall seismic evaluation of the HLP, respectively. The results of 3D evaluation presented in this paper allowed the authors to understand the behavior of the ore in soil-geomembrane interface within the heap during a seismic event.

Case study

The case study presented in this paper is a 120-m high HLP located at a mine site in northern Peru with a maximum capacity of almost 10 Mt. While the HLP was already been stacked with a capacity of 6 Mt, the authors were in charge of its stability verification, focusing on its seismic stability condition. In order to accomplish this, a large set of geotechnical field investigations and laboratory tests was carried out to characterize both the static and cyclic behaviour of the materials involved in the HLP design such as the soil foundation, soil-geomembrane interface of the liner system and the leached ore.

To evaluate the seismic stability of this HLP, several seismic analysis were performed which included preliminary pseudo-static slope stability analysis, 1D seismic response analysis, simplified calculations of SIPD and 2D and 3D dynamic analysis; the latter of these being described in this paper while the others are both described and compared by Regalado et al. (2016). Figure 1 presents a plan view and two representative cross-sections of the HLP. The following sections describe geotechnical characterization and 3D dynamic analysis details.

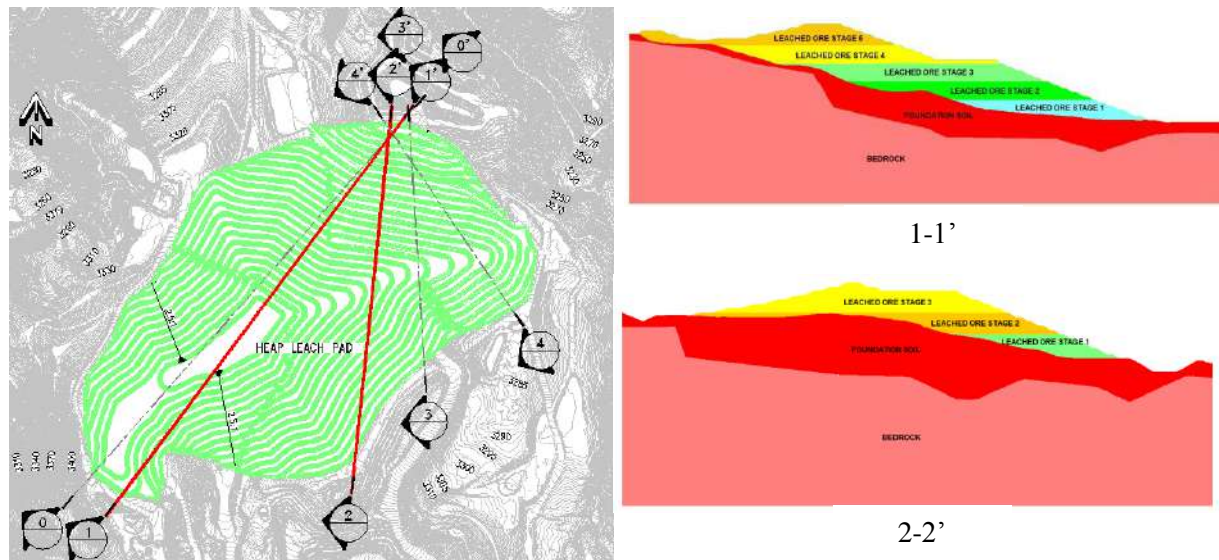


Figure 1: Plan view and cross-sections 1-1' and 2-2' of the heap leach pad

Field investigation and laboratory testing

The field work was focused on characterizing the foundations soils, soil-geomembrane interface and leached ore. Several samples of soil liner and geomembrane were collected in situ by removing part of the leached ore at the toe of the heap and cutting the geomembrane. On the other hand, leached ore samples were collected directly from the operating heap and their global of field particle-size distribution (PSD) curves, which included particles larger 3 in, were determined through several excavations along existing heap slopes. Additionally, several boreholes were executed at the toe of the heap to evaluate the

foundation over-consolidated clayey soils. Standard penetration tests (SPT) were executed and undisturbed samples were collected. No phreatic level was detected. Finally, a complete geophysical survey was completed along the heap and foundation soils.

Using the samples collected from the operating heap, a relatively large set laboratory tests were carried out. Regarding the clayey foundation soils, drained triaxial tests were carried out on undisturbed samples which, in conjunction with geophysical tests results, provide the information necessary for the analysis.

Leached ore was subjected to additional tests, since no database is available particularly for its dynamic properties. The samples collected were reconstituted in laboratory using the parallel gradation technique. This method scaled the field PSD curve to a parallel one considering the maximum particle size allowed by the testing device, which is usually between 10 to 15 times smaller the maximum particle size of standard LO and MW. This technique was first developed by Lowe (1964) and then extensively used by Marachi et al. (1969), Thiers and Donovan (1981) and Varadarajan et al. (2003) to perform drained monotonic triaxial tests on rockfill, crushed rock and alluvial soils, respectively. The PSD curve of the materials tested in the laboratory maintained the same coefficient of uniformity (C_u), PSD shape and relative density as the materials in the field but limiting the fines content to a maximum of 10%. Using this technique, monotonic drained triaxial tests were performed in a local laboratory in Lima, Peru. Additionally, the laboratory program included sets of special tests performed at the University of Texas at Austin using resonant column-torsional shear (RCTS) and cyclic-triaxial (CTX). The RCTS tests were performed in a sequential series on the same specimen with isotropic confining pressures (σ'_0) ranging from 200 kPa to 800 kPa. For each specimen, nonlinear RCTS tests were conducted at two or three σ'_0 over a shearing strain (γ) range from about 10^{-6} % to slightly more than 0.1%. CTX tests were conducted on these specimens at a single σ'_0 of 700 kPa for each specimen and over an estimated shearing strain range from about 0.01% to 1.4%. Further detail on these cyclic tests and others performed exclusively on leached ore and rock mine waste materials are presented by Parra et al. (2016).

Finally, two sets of large scale direct shear (LSDS) tests were performed on the low permeability soil-textured geomembrane interface: all of them tested on remoulded soil samples considering an interface consisting of the textured side an LLDPE 2.0 mm geomembrane in contact with a low permeability soil. One test was performed under normal stresses ranging from 100 to 800 kPa in a local laboratory and the other one was carried out at the TRI Environmental laboratory at Austin, Texas using normal stresses up to 2000 kPa, since most of the interface in the leach pad is subjected to normal stresses from 1000 to 2000 kPa. Along with the tests above described, a detailed review of all previous field and laboratory tests was executed that allowed to properly define both static and dynamic properties of all materials involved in the geotechnical design.

3D dynamic analysis

The HLP studied is located over an over-consolidated, unsaturated clayey soil foundation. Hence, only translational failures were of concern. Thus, the foundation soil was represented in all the analyses as a cluster with much higher strength than the interface or leached ore. The following sections briefly describe the static and dynamic geotechnical properties for evaluation as well as the analysis itself.

Static properties

The CD triaxial tests on leached ore provided nonlinear shear strength envelopes since it was considered cohesionless with a reducing friction angle as confining pressure increases. The logarithmic tendency developed by Leps (1970) for coarse granular materials was consistent with the results of CD triaxial tests, with a friction angle ranging from 35 to 39°. This nonlinear strength envelope was then used in the 3D analysis. On the other hand, the nonlinear monotonic stress-strain behavior was modeled using the Hardening Soil (HS) formulation (Brinkgreve et al., 2014). The HS is an advanced model for simulating the behaviour of different types of soil, both soft and stiff (Schanz, 1998). The HS formulation was calibrated with the resulting stress-strain curves of the CD triaxial tests. Another nonlinear shear strength envelope was defined for the interface and subsequently used in the 3D model. The studies published by Ayala and Huallanca (2014) and Parra et al. (2012) evidence the influence of the nonlinear behaviour of the interface for the stability analyses. The LSDS results at high normal stresses demonstrated the shear strength is nonlinear at high stresses. The Figure 2 shows the nonlinear shear strength envelopes for leached ore and interface.

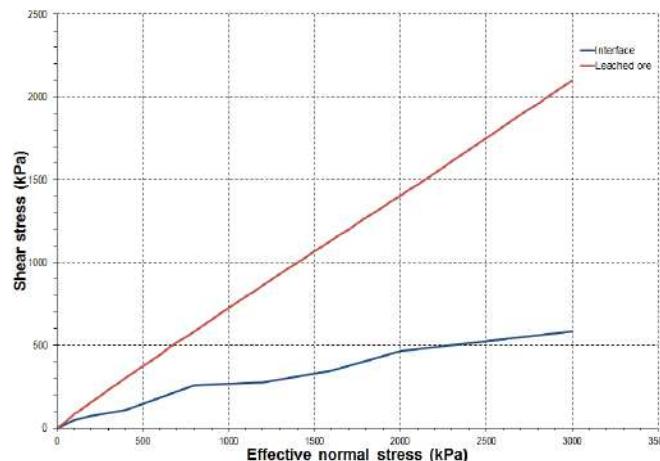


Figure 2: Nonlinear shear resistance envelope for leached ore and interface

Seismicity

The seismic analysis used the uniform hazard response spectrum for 100 years return period and defined for Class B soil (rock) as a design criterion. Seismic records from both horizontal components used as input for site response analysis were obtained from published motions from Peruvian subduction earthquakes recorded in Peru. The earthquake motions from the 1970 Lima and 2001 Atico were chosen to perform the dynamic the 2D and 3D analyses. It is important to mention that the both Lima and Atico earthquake motions were recorded near the epicenter of the event, capturing their high energy content. No other earthquake motions were selected due to the limited database available for Peru. These two seismic records were rotated to the most critical direction before any processing was done. Then, they were spectral matched to the 100 years return period uniform hazard response spectrum using the SeismoMatch software, which is based in the pulse wave algorithm proposed by Abrahamson (1992) and Hancock et al. (2006).

Dynamic properties

First, based on the curves obtained by the both RCTS and CTX tests on leached ore, the normalized shear modulus and damping ratio curves for this material were determined for confining pressures of 200 and 700 kPa. These proposed curves were compared with the Menq (2003) formulation, observing a good agreement from the small-strain range up to 0.01% of shear strain. Detailed discussion of these tests results is presented by Parra et al. (2016). Additionally, the geophysics survey results, performed directly on top of the heap's leached ore, were compared with the RCTS shear wave measuring obtaining a good agreement between the in-situ measurements and the predictions of the RCTS device. Figure 3 present the dynamic properties of the leached ore as tested in laboratory and as proposed for the seismic analysis.

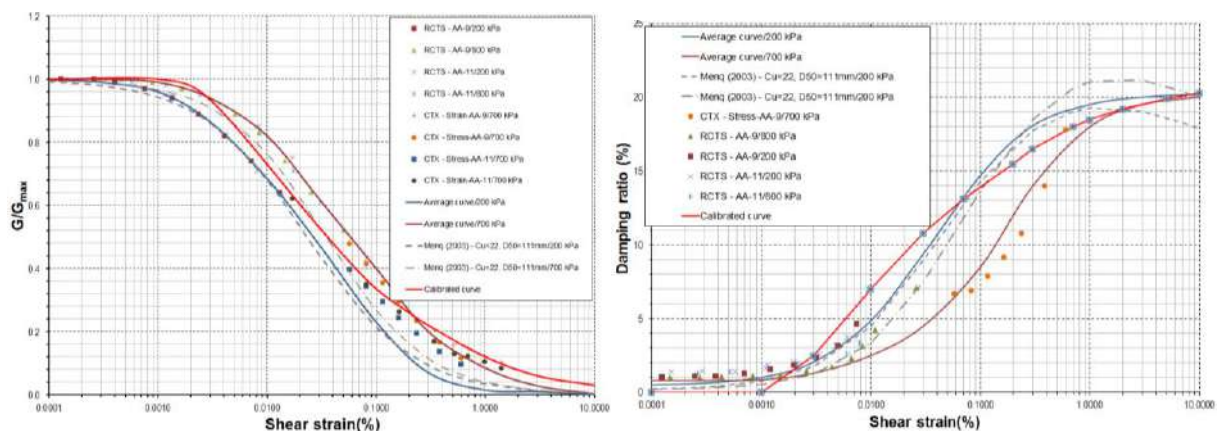


Figure 3: Normalized modulus reduction and damping ratio curves for leached ore.

The dynamic properties of the interface were defined by reviewing existing information on this matter. The backbone curve for the interface was modeled based on its static shear strength, according to the conclusion of Kavazanjian and Matasovic (1995) and Arab (2011). The damping ratio was modeled based on the cyclic shear tests on interfaces performed by Arab (2011) which show a relatively constant damping ratio value. This constant nature of the interface damping ratio is similar to the findings of Yegian et al. (1998), which only was used to determine the maximum shear modulus (G_{max}). It is important to mention that no cyclic shear test on the interface was developed for this paper; however, sensitivity seismic response analyses were carried out to analyse the inherent uncertainties of this modelling: these evaluations showed similar results for all cases.

For the foundation soil, the modulus reduction and damping ratio curves were represented by means of the Darendeli (2001) formulation, which use several parameters such as plasticity index (PI), over-consolidation ratio (OCR) and confining stress, among others. The Darendeli (2001) formulation was proposed by clayey and silty soils with a low percent of coarse grained soil. Additionally, the geophysical survey's shear wave velocity profiles of the foundation were used in the seismic analysis.

Finally, the dynamic properties of the bedrock were assigned considering an elastic material and only as a medium to broadcast waves. Because of the translational failure does not occur through this material, the shear strains induced by the earthquake were not of importance. Table 2 presents the main properties used in the seismic analysis.

Table 2: Main geotechnical parameters for seismic analysis

Material	Static properties			Dynamic properties	
	Cohesion (kPa)	Friction angle (°)	Shear modulus (MPa)	Maximum shear modulus (MPa)	Modulus reduction and damping ratio curves
Leached ore	Nonlinear envelope		Defined based on HS model calibration	Based on RCTS and geophysical tests	Based on RCTS and CTX tests
Soil-geomembrane interface	Nonlinear envelope		Nonlinear envelope	Based on Yegian et al. (1998)	Based on Kavazanjian and Matasovic (1995), Yegian et al. (1998) and Arab (2011)
Foundation soil	150	32	Defined based on HS model calibration	Based on geophysical tests	Darendeli (2001)

3D dynamic analysis

Modulus reduction and damping curves considerations

The dynamic analysis for the HLP was conducted using the Mohr-Coulomb's law of deformation stress. This law takes in considerations the energy dissipated and can determinate the damping rate as function of the shear plastic deformation. However, being an elastic-plastic model, the elastic branch does not consider the development of damping. In this case, *FLAC*^{3D} (Itasca, 2012) allowed implementing the Rayleigh damping model in addition to the mechanical damping hysteretic simplified of Mohr-Coulomb.

Modulus reduction and damping ratio curves for the leached ore and the soil foundation were calibrated using sigmoidal models implemented in *FLAC*^{3D}; Figures 4 and 5 show these calibrations. According to Yegian et al. (1998), for small accelerations transmitted in an interface model, the soil/geomembrane interface shows a rigid behavior; as the acceleration is increased, a sudden increase of displacement occurs. This could be attributed to a yield behavior of the interface (Yegian et al., 1998). Hence, interface elements in *FLAC*^{3D} using a Mohr-Coulomb model were used, since they can simulate the modulus reduction in a similar fashion as the Yegian et al. (1998) curve and exhibit damping ratio with levels up to 40%.

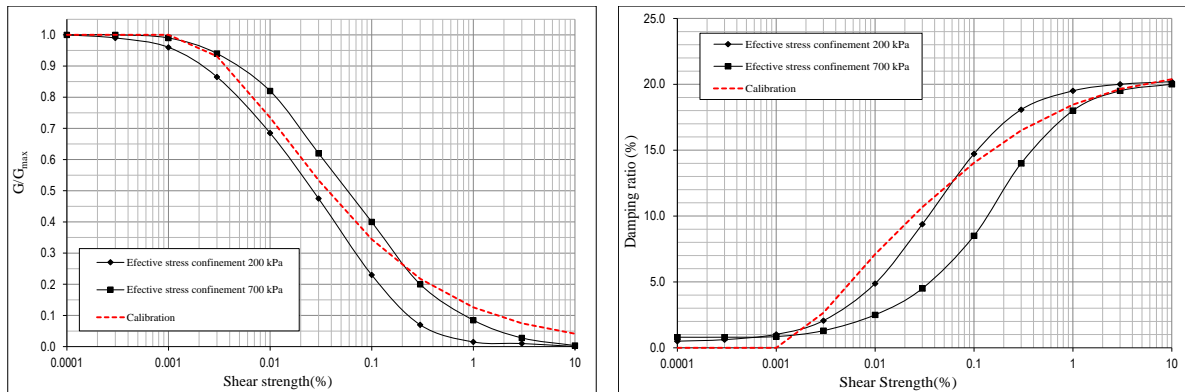


Figure 4: Normalized modulus reduction and damping ratio curves for leached ore.

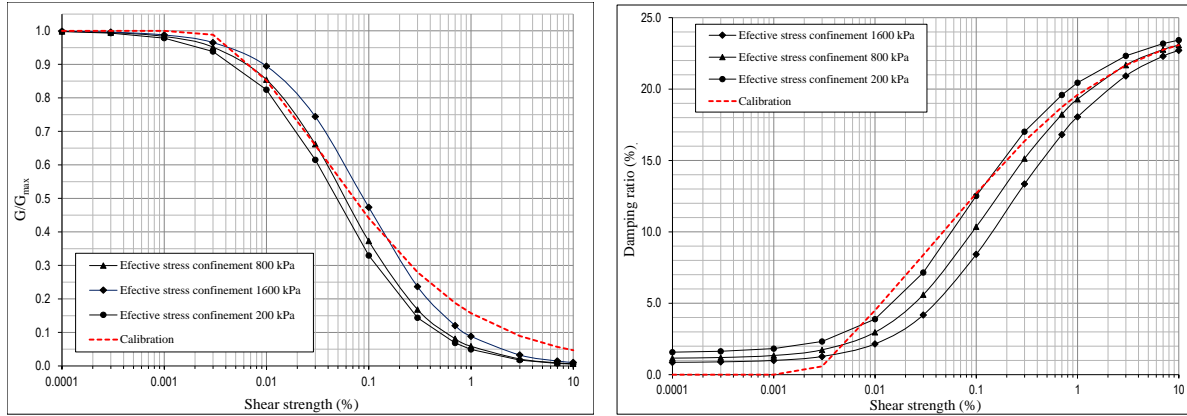


Figure 5: Normalized modulus reduction and damping ratio curves for soil foundation.

Model geometry

With the aim to simulate a proper transmission of energy of the seismic waves, the mesh of the model was defined using the Kuhlemeyer and Lysmer (1973) criteria, which defines the height (L) of the mesh zone as a function of the shear waves velocities (C_s) and the maximum transmittable frequency (f_s^{\max}), where zones with 5 m were defined for the leach ore and soil foundations materials, and 25 m for the rock, using the following expression.

$$L = \frac{C_s}{10 \times f_s^{\max}}$$

Table 2 indicates the maximum frequency transmitted by the mesh analysis for more critical shear waves, calculated with properties of each material at depths of 5 m. These frequencies are above of 4 Hz, which is the limit frequency of the seismic records used.

Table 2: Mesh size determination

Materials	Zone size m	Cs m/s	Freq. Cs Hz	Freq. Limit Hz
Foundation soil	5	320	6.4	4.0
Rock	25	1200	4.8	4.0
Leached ore	5	202	4.0	4.0

Figure 6 shows the final geometry and a representative cross-section of the model, as presented in *FLAC*^{3D}. As can be seen, with stages were considered for the heap geometry in order to represent its staged-construction. At each stage, vertical stresses and density were initialized according to the gravity, and horizontal stresses were calculated following the recommendations of Jaky (1994). It is important to mention that the model was rotated so that the North direction matched the one of the sliding direction of the 3D failure surface.

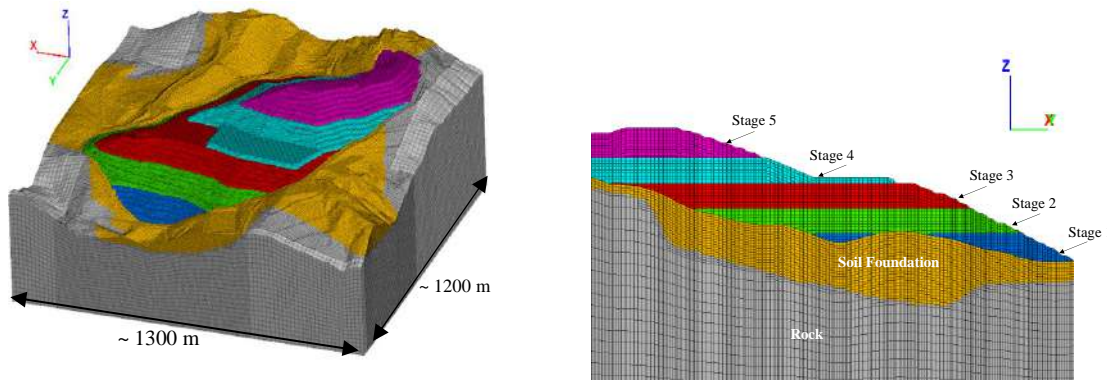


Figure 6: Geometrical model in *FLAC*^{3D}.

Static factor of safety

First, a static analysis was performed in order to obtain the stress distribution prior to the application of seismic records. Here, a stage-constructions analysis was included for the heap configuration. Then, the static equilibrium in dry conditions is obtained for the heap. Afterwards, a pore pressure grid generated by a groundwater level considered 5 m above the interface was incorporated to simulate the solution level above the leach pad. Figure 7 shows the total vertical displacements or settlements of the leached ore resulting from the staged-construction analysis, reaching a maximum settlement value of 1 m, which is considered acceptable given the height of the heap. It is also noted that the largest settlement occurs in the transition zone between each stage.

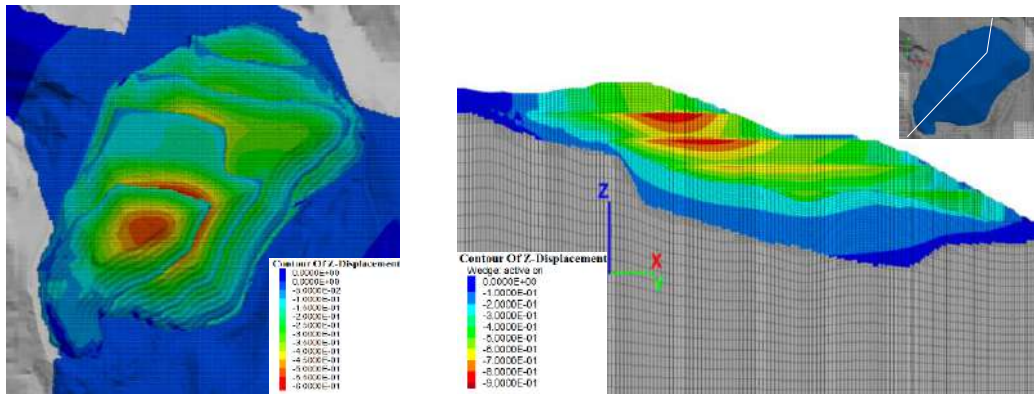


Figure 7: Heap settlement from static analysis in *FLAC^{3D}*

FLAC^{3D} provides a full solution of the coupled stress/displacement, equilibrium and constitutive equations. Given a set of properties, the system is determined to be stable or unstable. Using the shear strength reduction technique and by automatically performing a series of simulations while changing the strength properties, a factor of safety was defined. Figure 8 shows contours of static safety factor; the failure surface on the north of the heap being the one of concern. The minimum value is 1.425, covering isolated and small sectors, globally safety factors ranging from 1.6 to 1.7. This calculation was remarkably close to both the factor of safety and failure surface geometry determined by a 3D limit equilibrium analysis performed for the same HLP by Reyes et al. (2015).

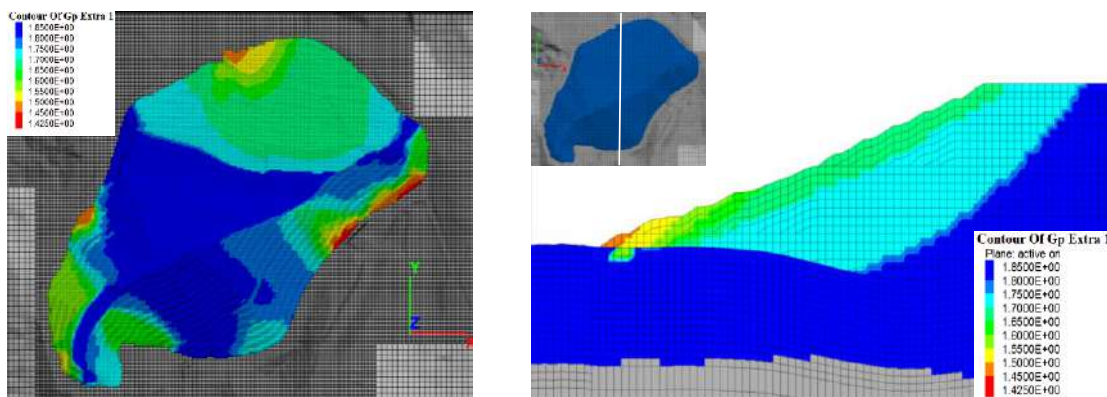


Figure 8: Static factor of safety in *FLAC^{3D}*.

Dynamic analysis of the base

Earthquake ground motions developed for dynamic analysis are usually provided as outcrop motions, normally rock outcrop motions. According to Mejia and Dawson (2006), the input register for a viscous and lineal base model is typically half of the original record. This represents a scaling factor of 0.5 for the original records; this procedure obviates the deconvolution process for rigid bases. To properly enter the seismic record in the viscous base, the earthquakes for this project were introduced as shear stress and as

a function of the shear wave velocity (C_s), the density (ρ) and the particle velocity of the upward propagation motion (v_{su}), according the following expression:

$$\tau = 0.5(-2 \times C_s \times \rho \times v_{su})$$

The procedure could be affected by changes in the surface topography of the rock model. A dynamic analysis was performed considering only the base (see Figure 9), in order to verify that the surface seismic records match with the design records. This verification was accomplished by comparing specific energy density, response acceleration spectra and the seismic record itself. The analysis showed that was necessary to apply scaling factors 1.206 and 1.364 for the 1970 Lima and 2001 Atico, respectively, in order to obtain the desired design earthquake.

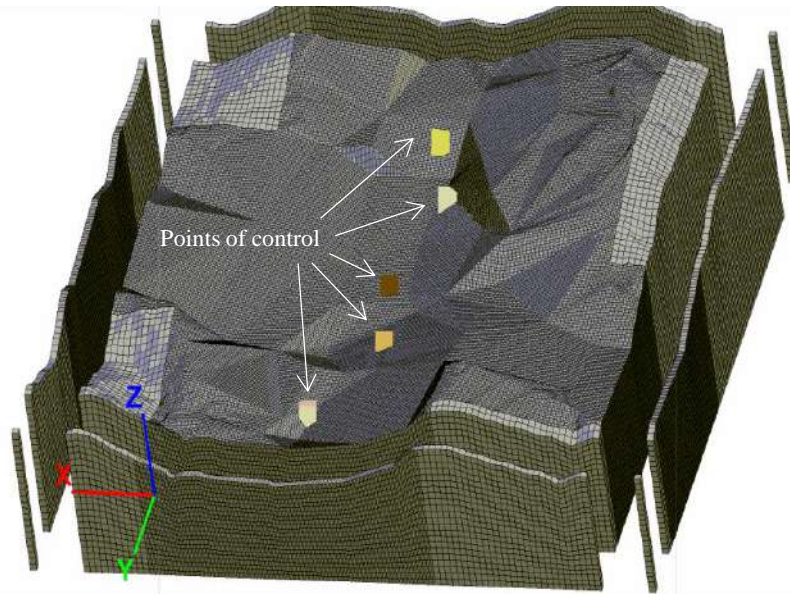


Figure 9: Base model and points of control

Elastic dynamic analysis

Figure 10 shows contours of the shear modulus and dynamic stiffness for the leached ore and interface, respectively. Using these parameters, an elastic undamped dynamic analysis was conducted in order to estimate the fundamental frequency of each material and for each seismic record. The obtained values were then used to implement a Rayleigh damping for each material in order to account for small-strain damping, which is not usually properly modeled the hysteretic damping of *FLAC*^{3D}. The Table 3 shows the values of fundamental frequency for each material.

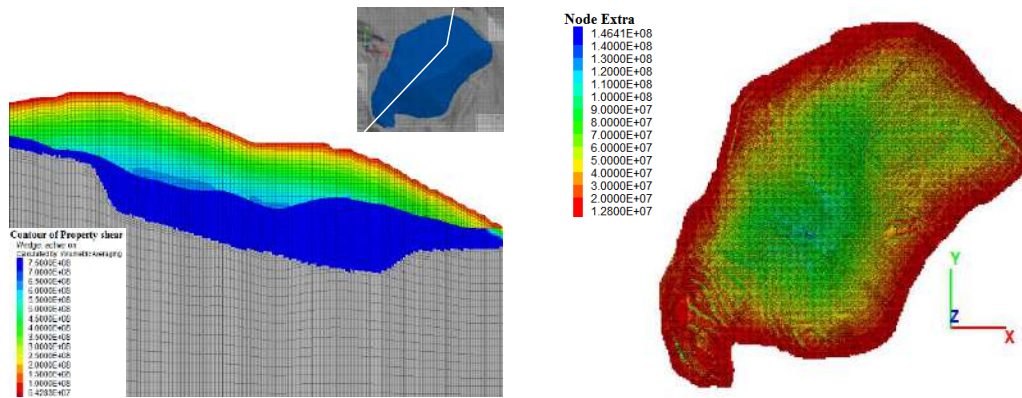


Figure 10: (Left) Dynamic shear modulus (Pa) of leached ore and (right) dynamic shear stiffness (Pa/m) of the interface

Table 3: Fundamental frequency of the materials

Seismic record	Fundamental frequency (Hz)		
	Rock	Foundation soil	Leached ore
1970 Lima	0.45	0.75	1.19
2001 Atico	0.52	0.52	1.35

Formal dynamic analysis

The formal dynamic analysis was performed considering the fully non-linear method, using *FLAC^{3D}*, to predict the dynamic behaviour of the heap under the seismic records described in previous sections. The analysis showed the stress/deformation behaviour of the HLP, especially in the interface. Figures 11 and 12 shows the shear displacements in the interface zone which were limited to 0.3 m in order to identify areas where is exceeded; note that Figures 11 and 12 only show the interface. Shear displacements above 0.3 m are highlighted in red. The largest displacements of the interface occur in areas of low vertical stress or low confinement, primarily at the toe of the HLP.

The horizontal earthquake was applied in the North-South direction which, due to a previous rotation of the model, matches the sliding direction of the HLP. Figure 13 shows the leached ore heap displacements and settlements for the 1970 Lima earthquake, where the heap displacement contours match the predicted failure surface geometry analysed by Reyes et al. (2015) in their 3D limit equilibrium analysis of the same HLP. Also, Figures 14 and 15 shows the shear strains of the leached for a critical section and in plan view, respectively. These deformations were limited to 3.5%, as recommended by Ishihara (1996). The sectors with higher shear strains cover about 20 m and are located at the toe of the

heap in areas of low confinement. Finally, Figure 16 shows control points located at the interface while Figure 17 shows the development of shear strains at the interface during the earthquake motions.

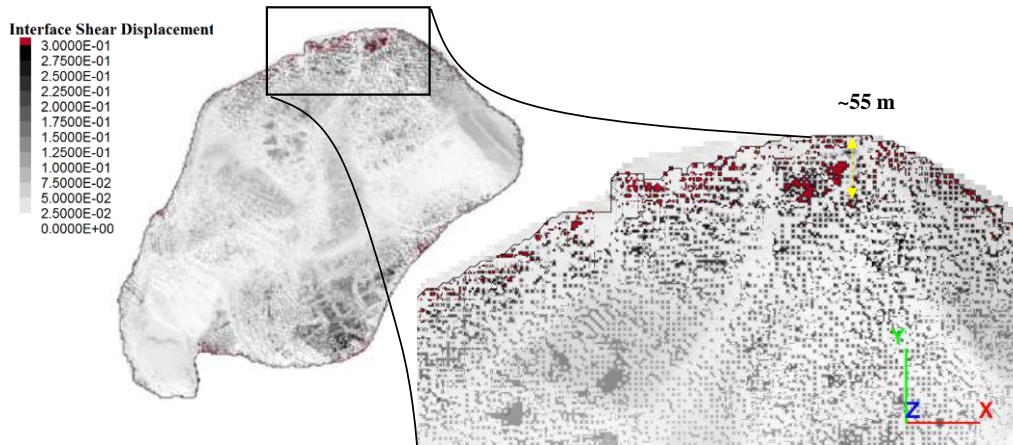


Figure 11: Shear displacements in the interface, seismic record 1970 Lima

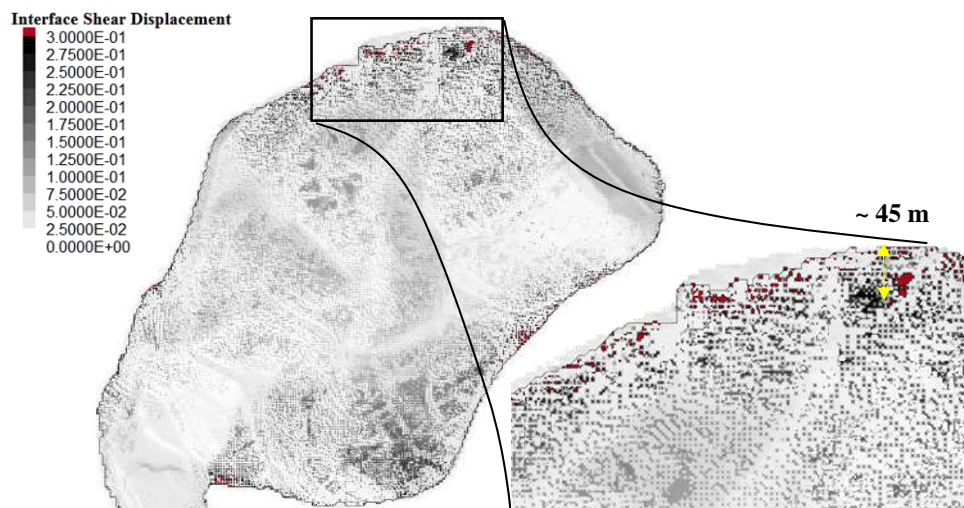


Figure 12: Shear displacements in the interface, seismic record 2001 Atico

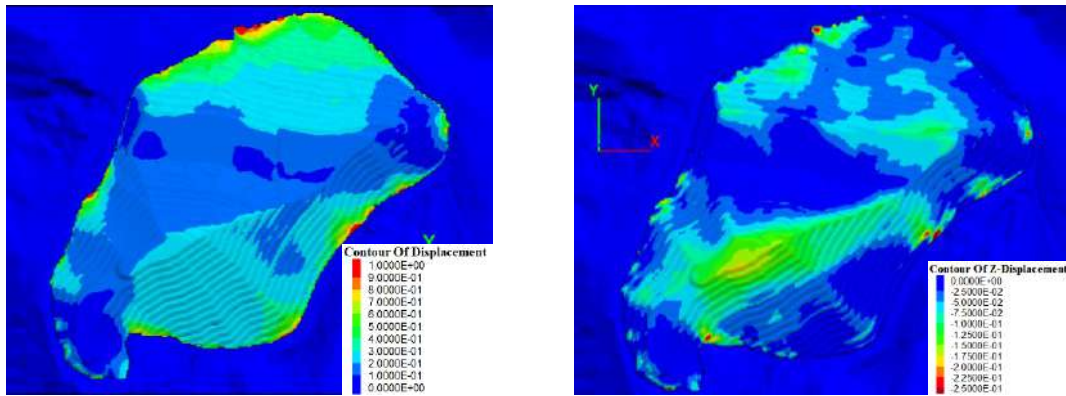


Figure 13: (Left) Total displacements and (right) vertical displacements on the heap surface from 1970 Lima

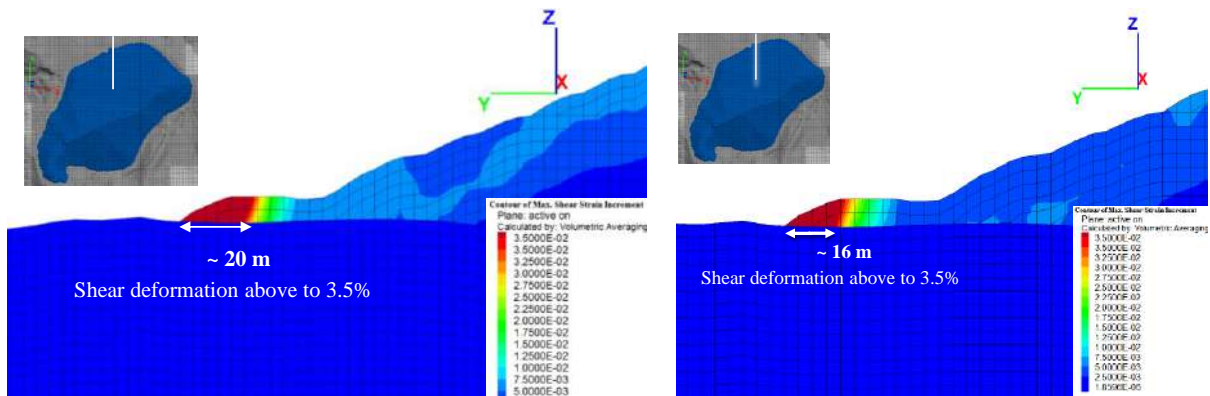


Figure 14: Maximum shear strain of leached ore for at the toe of the heap for the (left) 1970 Lima and (right) 2001 Atico earthquakes

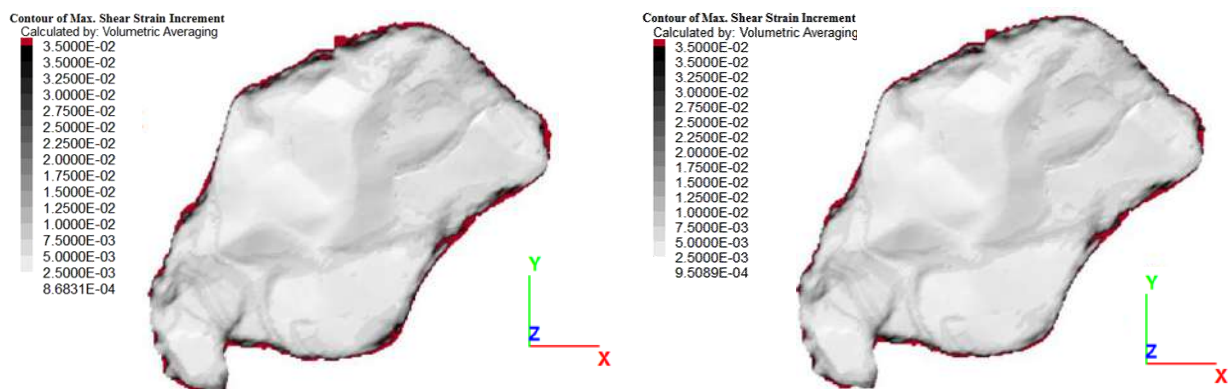


Figure 15: Plan view of the maximum shear strain of leached ore for the (left) 1970 Lima and (right) 2001 Atico earthquakes

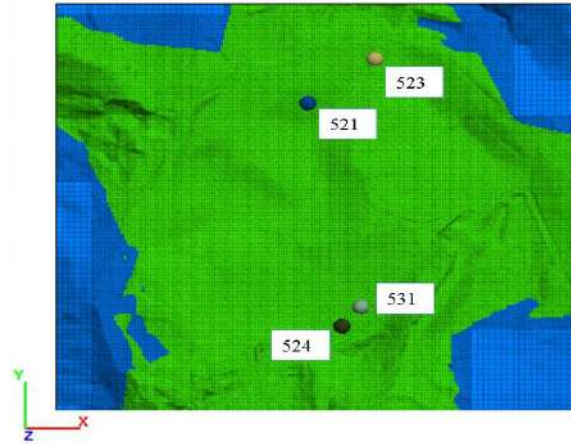


Figure 16: History locations on the interface, plan view.

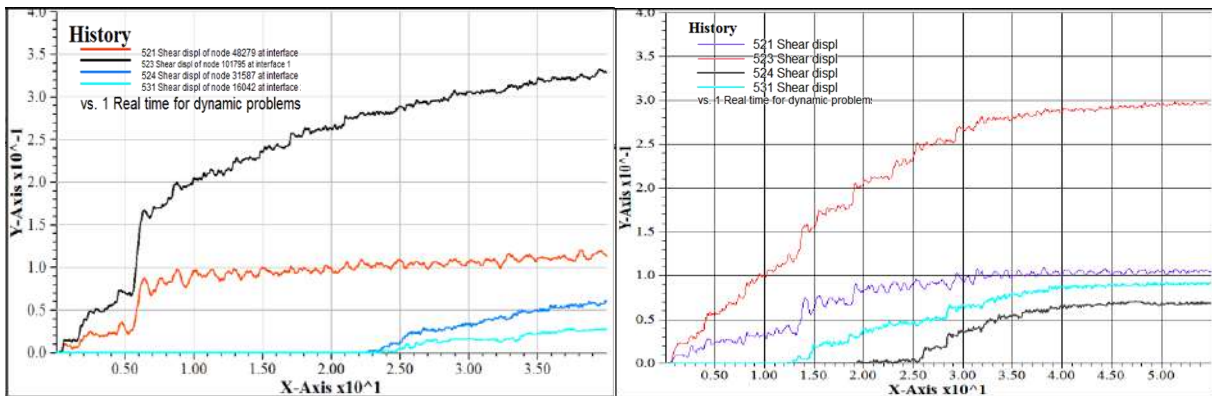


Figure 17: Shear strain development for control points in the interface for the (left) 1970 Lima and (right) 2001 Atico earthquakes

The results show that the highest shear displacements and the highest leached ore shear strain levels developed mostly around the toe of the heap in areas with low confinement. The low damping ratio of the leached ore, cohesionless nature of the ore and low confinement were found to induce both high displacements of ore and within the interface. Since the focus of the analysis is calculating shear displacements of the soil-geomembrane interface, the analysis allowed the authors to assess their development within the leach pad and to further define the seismic design of the HLP. Regalado et al. (2016) describe in the detail how these results were compared with simplified calculation of seismic induced permanent displacements and 2D dynamic analysis in 2 cross-sections of the HLP and how apparent instabilities defined by these evaluations were overcome using the 3D results.

Conclusions

The dynamic behavior of a Peruvian heap leach pad was analyzed for two representatives spectrally matched earthquakes, 1970 Lima and 2001 Atico, using the computer program *FLAC*^{3D} in order to validate its seismic design. Several geotechnical investigations, advanced laboratory tests, a proper geometrical construction considering the power transmission concepts, and proper definition of static and dynamic properties of materials was performed for the 3D model.

The static analysis showed that surface settlements are within an acceptable, considering the final height of the pad. In addition, local and global safety factors, which consider the three-dimensional effect, are acceptable according to general criteria used in similar projects and very close to a three-dimensional limit equilibrium slope stability analysis performed for the same heap by Reyes et al. (2015).

The analysis focused on the behaviour of the interface since translational failures were of concern. Shear displacements and strains were monitored both in the interface and within the heap. The shear displacements in the interface resulted in values mostly lower than 0.3 m, with local areas at the toe of the heap with values. The low damping ratio of the leached ore, cohesionless nature of the ore and low confinement were found to induce both high displacements of ore and within the interface. However, these sectors were found to do not represent a threat to the overall stability of the heap and integrity of the liner system. A comparison of the results presented in this paper and extended description of the seismic design of the studied heap leach pad is presented by Regalado et al. (2016).

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