# *Procedures for estimating seismic permanent displacements on tailings storage facilities and mine waste dumps*

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Even though the pseudo-static procedure for seismic slope stability analysis of tailings storage facilities (TSF) and mine waste dumps (MWD) is very common in practice, new methodologies have been studied regarding the calculation of seismic induced permanent displacements of earth structures, which is considered a more reliable parameter for evaluating the seismic stability of a geotechnical structure compared to the calculation of a factor of safety (FOS).

The authors have studied the application of several approaches to assess the seismic stability of TSF and MWD based on two case studies. One-dimensional (1D) nonlinear seismic response and slope stability analyses were performed as part of these applications. Cyclic laboratory tests on coarse tailings and mine waste and the dynamic curves derived were compared to current state-of-practice literature curves.

The resulting displacements showed, in general, a good correlation between the procedures evaluated. This research suggests that the analysis should be less focused on the pseudo-static FOS as a parameter to predict the seismic stability of TSF and MWD, unless a rational criterion is chosen for the determination of the seismic coefficient.

Keywords: slope stability, response analysis, cyclic laboratory tests

## 1 INTRODUCTION

Historically, most civil engineering structures built in Peru are design to endure strong seismic events expected in this region. These events are caused by the subduction zone of the Nazca plate beneath the Sudamerican plate. Several local studies in Peru, such as Castillo & Alva (1993) and Gamarra & Aguilar (2009), support the probability of strong earthquakes which leads to a heavily oriented seismic design. Those researches show isoaccelerations maps for different return periods and soil types; however, the Peruvian mining authority typically requests site seismic hazard assessments for each mine site.

During the last decade, TSF in Peru are currently designed without risking any slope failure that may involve soil liquefaction as a part of its mechanism. Therefore, extreme flow failure or a significant drop of the materials shear strength is prevented. Hence, TSF seismic stability is carried out through the pseudo-static approach using a seismic coefficient ranging from 1/2 to 2/3 of the peak ground acceleration (PGA). A similar approach is taken when designing MWD, whether failures through the mine waste or the foundation are expected. Only in particular cases, seismic induced permanent displacements (SIPD) are calculated usually using the Newmark (1965) or Makdisi & Seed (1978) methods for both structures. However, modern criteria for seismic design of earth structures define a maximum allowable displacement these structures can sustain. As a consequence, methods such as the Bray & Travasarou (2007) are being used to estimate SIPD through a simplified coupled procedure. Furthermore, the Bray & Travasarou (2009) method allows to select a seismic coefficient based on maximum allowable displacements for any particular structure, thus improving the pseudo-static approach.

Among mining structures, TSF are considered sensitive to SIPD since most of these facility's designs are focused on retaining fine tailings supported by dikes made of waste rock or coarse tailings materials. Seismic induced permanent displacements of 100 to 200 cm are considered allowable limits based on its freeboard; any failure or dike overtopping can lead to heavy environmental and economic damage, as well as life losses. Subsequently, a great deal of effort is put on whether determining an appropriate seismic coefficient to use on a pseudo-static analysis or estimating reliable values of SIPD. Similarly, MWD are usually subjected to equivalent evaluations where failures along the mine waste contained and/or the foundation are analyzed. The objective of this paper is to compare different approaches employed in practice to determine SIPD for TSF and MWD and their related calculations. First, assessing the dynamic response of these materials based in the current state of art one-dimensional (1D) nonlinear seismic response analysis with the use of Deepsoil software (Hashash, 2014) and, lastly, calculating SIPD with the Houston et al. (1987), Makdisi & Seed (1978) and Bray & Travasarou (2007) methods

## 2 THEORETICAL BACKGROUND

Kramer (1996) suggested two approaches to deal with seismic stability analysis: inertial stability and weakening stability analysis. The first one may be used in TSF and MWD as long as soil liquefaction is not expected to be involved in their slope failure mechanism. Inertial stability analysis deals with displacements produced by temporary exceedances of the material's shear strength by dynamic stresses, assuming that this shear strength remains relatively constant during the seismic event. A FOS calculation by a pseudo-static analysis and SIPD calculation are methodologies used to understand inertial instability.

By an extensive review of existing methods, Murphy (2010) defined three approaches to estimate seismic induced displacements: rigid-block, decoupled and coupled analysis. For the rigid-block analysis, the Newmark (1965) method is the most recognizable; the Makdisi & Seed (1978) is one of the most used decoupled methods; and the Bray & Travasarou (2007) as well as numerical dynamic analyses performed by software such as PLAXIS or FLAC are part of the coupled methods.

In order to tackle the methods mentioned above, a review of the seismic response of the materials involved with the analysis should be accounted, mainly for coarse tailings and mine waste. New information regarding dynamic behaviour of comparable materials is available and the current state of art of the seismic response analysis has greatly developed the last decade, as remarked by Stewart et al. (2008) and Hashash et al. (2010). The following sections describe the theoretical background of 1D seismic response analysis and SIPD calculations.

### 2.1 *1D seismic response analysis*

The surface seismic response of an earthquake is greatly influenced by site soil conditions. In order to quantify this, seismic response analyses are used to determine the dynamic soil behavior due to the shake of the rock immediately beneath it (Kramer, 1996). To quantify the seismic response of a rock, seismic hazard studies are performed. Dynamic behavior of rock is less influenced by the earthquake nature due to its large stiffness. 1D seismic response analyses are based on the hypothesis that all the soil boundaries are horizontal and that soil response is particularly affected by seismic shear waves, whose propagation turns vertical as it approaches the surface.

The analysis methodology depends on how the soil behavior is modeled. A linear method (LM) analysis relies on the use of transfer functions in the frequency domain. However, the nonlinear behavior of soils, which contrasts with the linear assumption of the LM approach, makes this methodology quiet restricted. In order to account for such restrictions, a simple iterative process involving dynamic equivalent linear properties of soil can be used; this methodology is called the equivalent linear method (ELM). As mentioned, this methodology is still linear up to some extent since it focuses on searching the elastic parameters of the soil. These parameters should be consistent with seismic induced shear strain levels for each soil layer involved in the analysis.

A fully nonlinear analysis (NLM) is capable of modeling the hysteretic behavior of soils due to earthquake loading. It uses a direct numerical integration in the time domain. Through this analysis, a linear or nonlinear stress-strain relationship can be followed by a number of small incremental linear steps. Such relationship is generally modeled by a hyperbolic model.

The load, unload and reload conditions, generally known as the extended 4 Masing (1926) rules, of the soil under cyclic loading was observed and proved by Matasovic (1993b) using the DMOD (Matasovic, 1993a) software. Currently, Hashash et al. (2010) has greatly improved the deficiencies encountered when using the NLM approach (Stewart et al., 2008) by the development of the DeepSoil software (Hashash, 2014).

## 2.2 *Seismic induced permanent displacements*

### 2.2.1 *Newmark (1965) and Houston et al. (1967)*

Newmark (1965) was the first to formulate the rigid-block analogy, and his methodology has been widely used to calculate SIPD for most geotechnical structures. The Newmark method considers a rigid block mass sliding on an inclined plane, whose SIPD equals the double integration of the difference between earthquake acceleration and a yield acceleration (ky), with the latter concept referring to the overall slope resistance, which in turn depends primarily on the dynamic strength of the material along the critical sliding surface and the structure's geometry and weight (Bray, 2007). Several authors have modified the original Newmark (1965) method to overcome simplifications such as the inclined plane and the rigidity of both the sliding mass and slip surface assumptions.

Houston et al. (1987) modified the Newmark (1965) methodology by introducing a slip layer, whose "softened" properties would prevent accelerations within the sliding mass to exceed ky. Accelerations that surpass ky within the sliding mass would generate movements in it and then be inconsistent with the original assumption of the rigid-bock method. Typically, the seismic record below the slip layer is used to calculate displacements.

### 2.2.2 *Makdisi & Seed (1978)*

In their landmark paper, Makdisi & Seed (1978) formulated the decoupled method, which consists of two separate steps: a dynamic response analysis and a sliding response analysis. The first one is performed to quantify the accelerations experienced by the sliding mass. The second one is performed to calculate SIPD through double integration of an earthquake motion. Makdisi & Seed (1978) used average accelerations computed by the procedure of Chopra (1966) and sliding block analyses to compute SIPD of earth dams and embankments (Kramer, 1996).

Makdisi & Seed (1978) were the first to develop a series of calculation charts based on their simplified decoupled method by the analysis of three earthquake records with different magnitudes. One of their charts evaluates the seismic demand experienced by the sliding mass as a function of the slip surface depth, main body height, and crest peak acceleration of a dam. The other chart is employed to estimate SIPD with respect to the fundamental period of the embankment (Murphy, 2010). The Makdisi & Seed (1978) method is still widely used within the geotechnical community for a broad range of structures, primarily due to its simplicity, despite the fact that it was only developed for dams and embankments.

### 2.2.3 *Bray & Travasarou (2007)*

Bray & Travasarou (2007) presented a simplified coupled semi-empirical predictive model to estimate the SIPD based on the Newmark (1965) rigid-block method and numerical analysis, as a way to update the method developed by Makdisi & Seed (1978). This procedure involves a block failure model sliding over a nonlinear coupled surface (Rathje & Bray, 2000), which can represent the dynamic behavior of structures such as: dams, natural slopes, compacted fill dykes, and municipal solid waste fills (MSWF).

Bray & Travasarou (2007) noted that the major uncertainty for the evaluation of an earth structure is the seismic event. To overcome this issue, they took advantage of over 688 earthquake records and concluded that the spectral acceleration at a degraded period of the potential sliding mass is the most efficient and sufficient single ground motion intensity measure. The method captures the slope seismic resistance through its ky and initial fundamental period. Using these parameters as input, Bray & Travasarou (2007) presented formulations to estimate SIPD and to evaluate the probability of negligible SIPD. Finally, they showed that their estimates were generally consistent with 16 documented cases of earth dams and MSWF.

### 2.2.4 *Stress-strain analysis*

A powerful tool to estimate both static and SIPD is the use of stress-deformation analyses that employ 2-D and/or three 3-D finite elements or finite different models. These analyses include seismically-induced permanent strains in each element of the finite element mesh or zone of the finite difference model (Kramer, 1996). Conceptually, a fully-coupled nonlinear analysis should be able to calculate any SIPD in any slope; however, such analyses are very complex (Duncan & Wright, 2005). Without initial simplifying assumptions, the accuracy of the stress deformation analysis depends on the stress-strain or constitutive model capacity to represent the real soil behavior (Kramer, 1996). Computer programs such as PLAXIS or FLAC are widely used to assess the seismic behavior of most geotechnical structures.

## 3 CASE STUDIES GEOTECHNICAL OVERVIEW AND ANALYSES

The first case study presented is a 60-m high TSF with a global downstream slope of 1V:2.5H , as shown in Figure 1 (Pérez et al., 2015). The TSF is currently in its final configuration and is located over a medium hard rock. Its dike is composed of coarse tailings conventionally disposed by centrifugal equipment (cyclones); fine tailings are placed upstream. The coarse tailings dike is underlined by a gravelly drain and a pumping system keeps the beach as far as 300 m away from the crest. Piezometers and seepage analysis show a good agreement between them and support the good operation of the pumping system and the drain, guaranteeing a good water management surrounding the coarse tailings dike. These conditions support that no failure mechanism are expected to be triggered by coarse tailings liquefaction during an earthquake. Therefore, the application of 1D response analysis and SIPD methodologies previously assessed can be used, providing representative results.

The second case study is a 140-m high MWD that, due to limited suitable locations able to satisfy the minimum storage capacity required, forced to designers to place it on a wide valley, as shown in Figure 2. Previous geotechnical investigation showed that the foundation of this area was composed of large and heterogeneous deposits of alluvial and residuals soils that is over 80 m deep. Clayey, silty, sandy and gravelly soils were distributed all over its area. Consequently, during its design, this facility has been subjected to three-dimensional (3D) slope stability analysis (Reyes & Parra, 2014), short-term stress-strain and deformational assessments (Reyes & van Zyl, 2015) and detailed 1D nonlinear seismic response analyses (Reyes et al., 2015). As soil liquefaction of was expected only on reduced and isolated lenses of loose granular soils, SIPD were considered appropriate for the evaluation of the inertial seismic stability of the MWD.

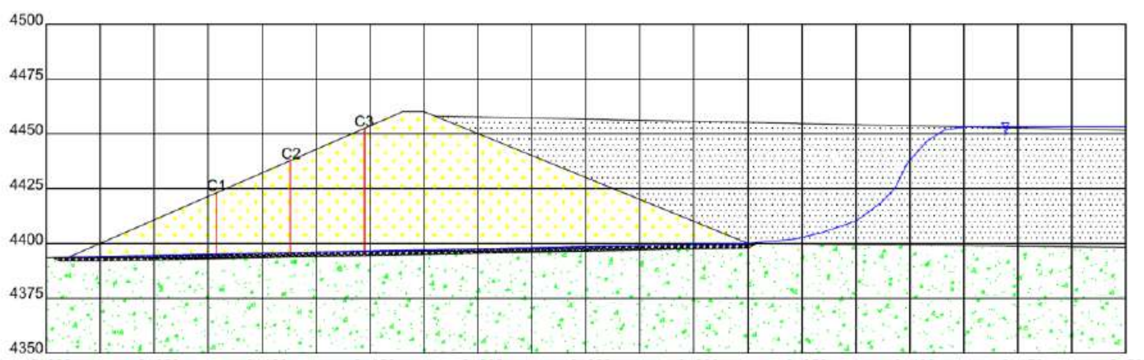

Figure 1. Critical cross-section of the TSF (Pérez et al., 2015)

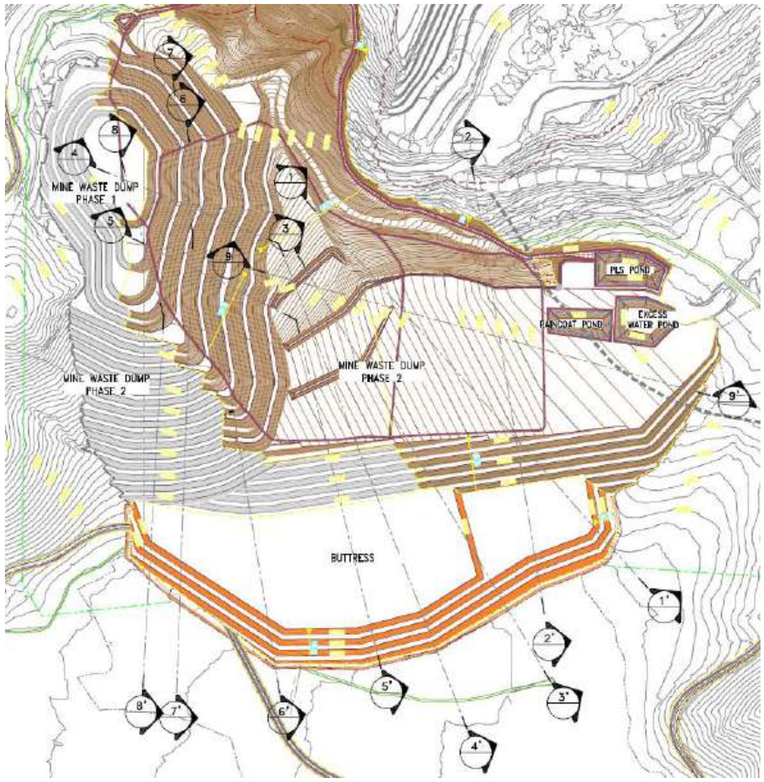

Figure 2. Plan view of the MWD (Reyes & van Zyl, 2015)

The following sections describe the geotechnical features and laboratory tests carried out on the coarse tailings and mine waste for both case studies, respectively. A detailed description of the geotechnical analysis performed for comparison purposes for this research is presented, which included 1D seismic response analysis and SIPD calculations using the Houston et al. (1987), Makdisi & Seed (1978) and Bray & Travasarou (2007) methods.

### 3.1 *Seismicity*

The uniform hazard response spectrums for 100 and 475 return periods (operation and closure conditions, respectively), from the site seismic hazard assessment were employed in all seismic evaluations. Seismic records from both horizontal components used as input for site response analysis were obtained from published motions from Peruvian subduction earthquakes recorded also in Peru. The earthquake motions from the 1974 Lima and 2001 Atico earthquakes were chosen. It is important to mention that the Lima and Atico earthquake motions were recorded near the epicenter of the event, capturing their high energy content. Other seismic records considered, such as the 2005 Tarapacá and 2014 Iquique motions, were recorded in Peru, far from their epicenters. As a consequence, low values of PGA and energy content were registered for these earthquakes, being subsequently discarded for the evaluations. No other earthquake motions were selected due to the limited database available for Peru. All 4 seismic records (two horizontal components per earthquake) were spectral matched to the 100 and 475 years return period response spectrums using the SeismoMatch software, which is based in the pulse wave algorithm proposed by Abrahamson (1992) and Hancock et al. (2006).

### 3.2 *Dynamic properties of coarse tailings and mine waste*

Coarse tailings dynamic properties were obtained from cyclic triaxial tests for confining pressures of 250 and 500 kPa. The shear modulus reduction and damping ratio curves were built out of 8 points, as shown in Figure 3. To extrapolate these results for shear strains from 1x10-3 to 10%, the data points were adjusted to a standard hyperbolic model.

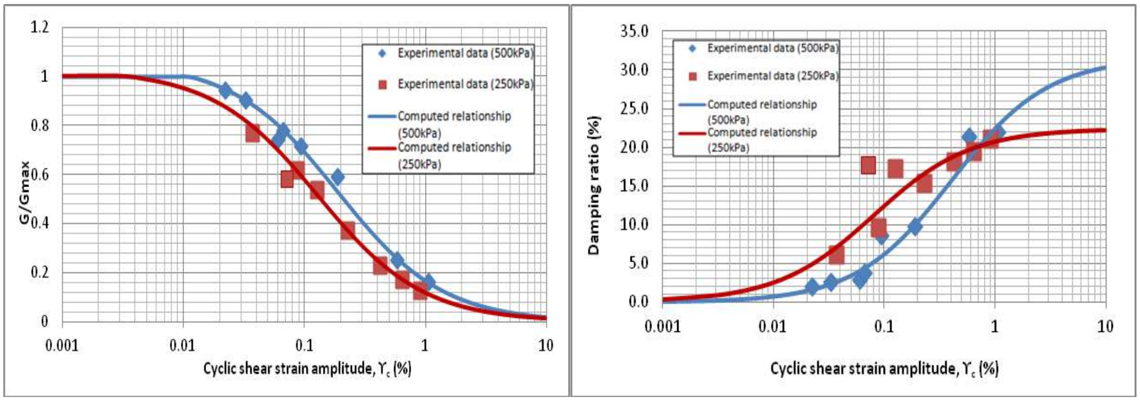


Figure 3. Cyclic triaxial test results and adjusted data for coarse tailings (Pérez et al., 2015)

The dynamic properties for the mine waste of the second case study were obtained from resonant column and torsional shear tests (RCTS) on confining pressures of 165 and 669 kPa. Due to the large size of the particles of the mine waste, a parallel gradation curve to the original particle size distribution was built. By doing this, the "scaled" sample had a maximum particle size of 3/4 inches and no scalping would be needed when tested on the RCTS device built in the University of Texas at Austin. This technique of parallel gradation was first developed by Lowe (1964), and then extensively used by Marachi et al. (1969), Thiers and Donovan (1981) and Varadarajan et al. (2003) to perform drained triaxial tests on rockfill, crushed rock and alluvial soils. In the last decade, many researchers, particularly Gesche (2002), De La Hoz (2007), Dorador (2010) and Ovalle et al. (2014), and practitioners such as Linero et al. (2007) and Palma et al. (2009) have used this technique when testing alluvial and waste rock materials. The shear modulus reduction and damping ratio curves were built out of 11 points, as shown in Figure 4. Similar to the coarse tailings case, the data points were adjusted and extrapolated using a hyperbolic model.

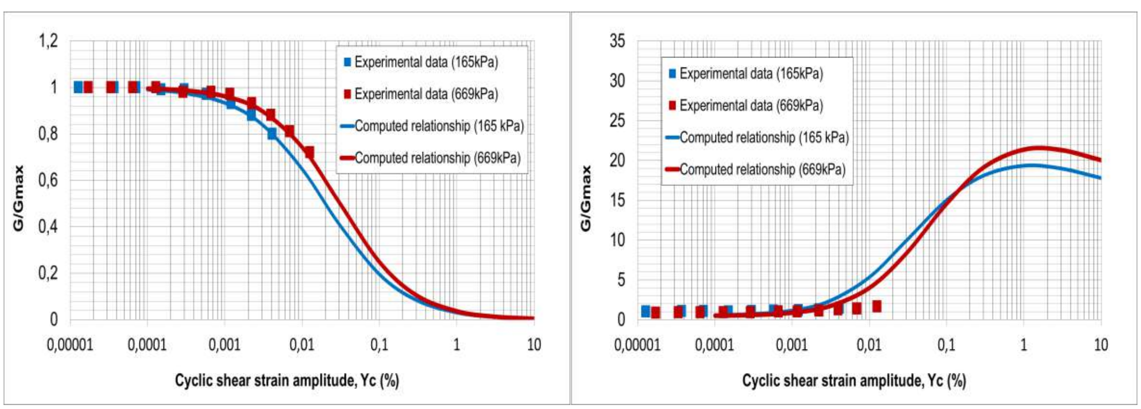


Figure 4. Resonant column and torsional shear test results and adjusted data for mine waste

It is important to mention that, to build a proper shear modulus reduction curve, the maximum shear modulus should be calculated through geophysical or resonant column tests since one of the limitations of cyclic triaxial testing is that the lowest shear strain to be tested is not low enough to adequately estimate the maximum shear modulus.

3.2.1 *Comparison of curves*

Prior to any calculation of SIPD, comparisons of the dynamic curves resulting from the cyclic tests described before were made with existing literature curves such as the ones proposed by Seed & Idriss (1970), Seed et al. (1986), Ishibashi & Zhang (1993), Darendeli (2001) and Menq (2003). These comparisons were made to determine which ones properly model the dynamic behaviour of coarse tailings and mine waste. Figure 5 shows the shear modulus reduction curves of the tested coarse tailings and the ones obtained from the Ishibashi & Zhang (1993), EPRI (1993) and Menq (2003) formulations for average confining pressures of 250 and 500 kPa. Similarly, Figure 6 shows the shear modulus reduction curves of the tested mine waste and the ones obtained from the Seed & Idriss (1970), Seed et al. (1986) and Menq (2003) formulations for average confining pressures of 165 and 669 kPa. These literature curves were selected due to its visual close fit with the tested material curves.

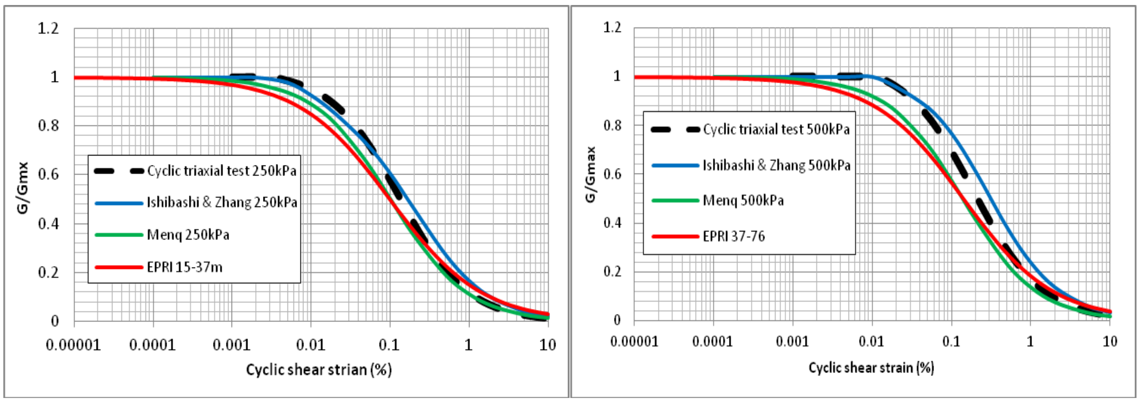


Figure 5. Shear modulus reductions curves comparison for coarse tailings (Pérez et al., 2015).

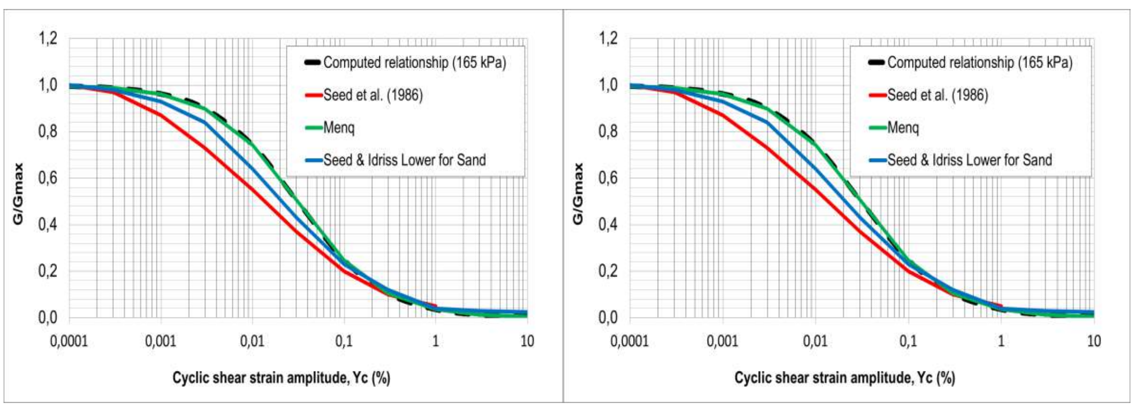


Figure 6. Shear modulus reductions curves comparison for mine waste.

In order to extend the comparison and to define which literature-based curve results in a response spectrum similar to the one obtained when using the cyclic tests results, one-layer soil columns analysis were built to perform ELM seismic response analysis. This method was preferred over the NLM because the last one would have required a detailed discretization of the soil column and, in consequence, different dynamic curves for different confining pressures. However, the ELM approach provides the same results whether a discretization is performed or not.

The results showed that the curves from the Ishibashi & Zhang (1993) and Menq (2003) formulations result in the closest response spectrum to the one calculated using the cyclic triaxial test results for the coarse tailings. However, the Menq (2003) curves are preferred for this material and subsequently used in this paper due to its best behavior as the confining pressure increases; the Ishibashi & Zhang (1993) curves often encounter problems for high confining pressures. The Seed & Idriss (1970), Seed et al. (1986) and Menq (2003) formulation resulted in close fits for the mine waste case. Similar to the coarse tailings, the Menq (2003) curves are preferred due to its varying nature when increasing the confining pressure. In general, the Menq (2003) curves were used for both cases, given that appropriate parameters were used for their formulations.

### 3.3 *Seismic induced permanent displacements calculations*

SIPD were calculated for both return periods (100 and 475 years) using the Makdisi & Seed (1978), Houston et al. (1987) and Bray & Travasarou (2007). The critical section showed in Figure 1 was used for the TSF and the cross-section 6 showed in plan view in Figure 2 was used for the MWD.

NLM seismic response analyses were used for the Houston et al. (1987) calculations using the software DeepSoil (Hashash, 2014) and D-MOD (Matasovic, 1993a). For the case of Bray & Travasarou (2007) analysis, representative response spectra were used, considering free field conditions (not taking into account the facilities). For the particular case of the MWD, seismic response spectra which considered the seismic behavior of the foundation soils of section 6 was used, which were assessed in detail by Reyes et al. (2015).

Table 1 shows the results of the SIPD developed along the failure surface of the TSF. As can be seen, there is a general agreement between the results of the Houston et al. (1987) and Bray & Travasarou (2007) methods, with the latter showing not only a rationally conservative range but also predicting a non-existing probability of negligible SIPD. On the other hand, Makdisi & Seed (1978) underestimates the displacements when compared with the other method for the 100 year return period event; the opposite happens when observing the results of the 475 years return period event, where the displacements are relatively overestimated.

Table 2 shows relatively similar comparison results for the MWD as Table 1. Very close results are predicted by the Houston et al. (1987) and Bray & Travasarou (2007) methods. The Makdisi & Seed (1978) method, again, underestimates the results for the 100 year return period, respectively, when compared to the other results. Nevertheless, the 475 year return period results for this last calculation agrees with the other methods.

Table 1. Seismic induced permanent displacements obtained for the tailings storage facility.

| Return period (years) | Seismic record | Yield acceleration (g) | Makdisi & Seed (1978) | Houston et al. (1987) | | Bray & Travasarou (2007) | | |
|---|---|---|---|---|---|---|---|---|
| | | | Value (cm) | Value (cm) | Average (cm) | PND (%) | Average (cm) | Range (cm) |
| 100 | Lima | 0.05 | 4-30 | 42 | 37 | 0 | 23 | 12-46 |
| | Atico | | 4-28 | 32 | | | | |
| 475 | Lima | | 170-400 | 175 | 135 | 0 | 92 | 46-184 |
| | Atico | | 110-290 | 94 | | | | |

PND: Probability of negligible displacements

Table 2. Seismic induced permanent displacements obtained for the mine waste dump.

| Return period (years) | Seismic record | Yield acceleration (g) | Makdisi & Seed (1978) | Houston et al. (1987) | | Bray & Travasarou (2007) | | |
|---|---|---|---|---|---|---|---|---|
| | | | Value (cm) | Value (cm) | Average (cm) | PND (%) | Average (cm) | Range (cm) |
| 100 | Lima | 0.04 | 4-10 | 16 | 13 | 0 | 10 | 5-19 |
| | Atico | | 4-9 | 9 | | | | |
| 475 | Lima | | 80-125 | 92 | 90 | 0 | 73 | 37-146 |
| | Atico | | 70-100 | 88 | | | | |

PND: Probability of negligible displacements

## 4 CONCLUSIONS

Several procedures for estimating SIPD and performing seismic response analysis were evaluated. The rigid-block Houston et al. (1987), Makdisi & Seed (1978) decoupled and Bray & Travasarou (2007) coupled procedures were reviewed and compared.

Existing literature shear modulus reduction and damping ratio curves were compared to project-specific cyclic laboratory tests on coarse tailings and mine waste. Visual a seismic response-based comparisons showed that Menq (2003)'s curves can represent the dynamic properties of both coarse tailings and mine waste, given that appropriate parameters are used as input for his hyperbolic formulation. More testing is needed to extend this conclusion to different coarse tailings and mine waste than the ones tested for this research.

The procedures reviewed for estimating SIPD were used and compared in two case studies of a TSF and MWD, using project-specific dynamic properties and details. The results showed, in general, a fair to good agreement between them. The Houston et al. (1987) were within the range estimated by the Bray & Travasarou (2007) method for all return period seismic events evaluated and for both facilities. In contrast, the Makdisi & Seed (1978) method proved to, in general, underestimate and overestimate the results for the 100 and 475 year return period, respectively, when compared to the other methods. This conclusion is in agreement with the findings of Bray & Travasarou (2007), who showed consistency between the results of their method when compared to observed seismic displacements on earth dams and MSWF and concluded that the Makdisi & Seed (1978) method can yield both conservative and unconservative displacements.

Reyes & Pérez (2015) also performed a similar comparison for a specific case study of heap leach pad (HLP) and concluded the same regarding the differences and similarities of the procedures described above. Given these findings, the authors recommend the use of the Bray & Travasarou (2007) method for the inertial seismic analysis of TSF, MWD and HLP, since it involves relatively simple calculation in in comparison the numerical complexity of Newmark (1965) type analysis or the use finite-element finite-difference models and yields rationally conservative results. However, it is important to mention that SIPD are sensitive to the fundamental period of the sliding mass and correspondent spectral acceleration, which are inputs for the Bray & Travasarou (2007) procedure. Therefore, the determination of the dynamic characteristics of the materials involved in the sliding mass and the correct selection of response spectra for design are critical. This research suggests that the inertial seismic design of mining earth structures can be focused on determining SIPD rather than simply determining pseudo-static FOS, unless rational criteria are used to define the seismic coefficient, such as the one presented by Bray & Travasarou (2009).

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