

# Simplified calculation of seismic displacements on tailings storage facilities

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## ABSTRACT

Even though the pseudo-static procedure for seismic slope stability analysis of tailings storage facilities (TSF) is very common in practice, new methodologies have been studied regarding the calculation of seismic induced permanent displacements of earth structures, which is considered a more reliable parameter for evaluating the seismic stability of a geotechnical structure compared to the calculation of a factor of safety. In general, there are three approaches for calculating the seismic response analysis of earth structures: (1) rigid-block, (2) decoupled and (3) coupled procedures. The Houston et al. modification of the Newmark procedure is the most recommended rigid-block procedure. The Makdisi and Seed decoupled simplified procedure is one of the most popular among practitioners and is also based on the Newmark method. Recently, Bray and Travarasrou updated the Makdisi and Seed method, analyzed new information by using a numerical coupled analysis and developed a simplified coupled procedure that may be used more consistently. As for coupled procedures, software such as PLAXIS and FLAC have been used as the most accurate tools in geotechnical engineering for estimating seismic induced permanent displacements.

The authors have studied the application of these approaches to assess the seismic stability of TSF based on a case study. 1D nonlinear seismic response and slope stability analyses were performed as part of these applications. Dynamic laboratory tests on tailings and the dynamic curves derived were compared to current state-of-practice literature curves.

The resulting displacements showed, in general, a good correlation between these procedures. This research suggests that the analysis should be less focused on the pseudo-static factor of safety as a parameter to predict the seismic stability of a TSF, unless a rational criterion has been chosen for the determination of the seismic coefficient. Finally, the Bray and Travarasrou procedure is suggested as a rational method to properly estimate seismic induced permanent displacements for TSF as a better index to establish the seismic stability of these facilities.

**Keywords:** seismic induced permanent displacements, tailings storage facility, response analysis

## INTRODUCTION

Historically, most civil engineering structures built in Peru are design to endure strong seismic events expected in this region. These events are caused by the subduction zone of the Nazca plate beneath the Sudamerican plate. Several local studies in Peru, such as Castillo and Alva (1993) and Gamarra and Aguilar (2009), support the probability of strong earthquakes which leads to a heavily oriented seismic design. Those researches show isoaccelerations maps for different return periods and soil types; however, the Peruvian mining authority typically requests site seismic hazard assessments for each mine site.

During the last decade, most TSF are currently design without risking any slope failure that may involve soil liquefaction as a part of its mechanism, therefore, extreme flow failure or a significant drop of the materials shear strength is prevented. Hence, TSF seismic stability are carried out through the pseudo-static approach using a seismic coefficient ranging from 1/2 to 2/3 of the peak ground acceleration (PGA). Only in particular cases, seismic induced permanent displacements (SIPD) were calculated usually using the Newmark (1965) or Makdisi and Seed (1978) methods. However, modern criteria for seismic design of earth structures defines a maximum allowable displacement those structures can sustain. As a consequence, methods such as the Bray and Travararou (2007) are being used to estimate SIPD through a simplified coupled procedure. Furthermore, the Bray and Travararou (2009) method allows to select a seismic coefficient based on maximum allowable displacements for any particular structure, thus improving the pseudo-static approach.

Among mining structures, TSF are considered sensitive to SIPD since most TSF designs are focused on retaining fine tailings supported by dikes made of waste rock or coarse tailings materials. Seismic induced permanent displacements of 100 to 200 cm are considered allowable limits based on its freeboard; any failure or dike overtopping can lead to heavy environmental and economic damage, as well as life losses. Subsequently, a great deal of effort is put on whether determining an appropriate seismic coefficient to use on a pseudo-static analysis or estimating reliable values of SIPD. The objective of this paper is to compare different approaches employed in practice to determine SIPD for TSF and its related calculations. By first assessing the dynamic response of these materials based in the current state of art one-dimensional (1D) nonlinear seismic response analysis with the use of Deepsoil software (Hashash, 2012) and, lastly, calculating SIPD with the Houston et al. (1987), Makdisi and Seed (1978) and Bray and Travararou (2007) methods.

## THEORETICAL BACKGROUND

Kramer (1996) suggested two approaches to deal with seismic stability analysis: inertial stability and weakening stability analysis. The first one may be used in TSF as long as soil liquefaction is not expected to be involved in the TSF slope failure mechanism. Inertial stability deals with displacements produced by a temporary exceed of dynamic loading that is greater than soil dynamic strength, assuming that the latter remains relatively constant during the seismic event. Either a factor of safety calculation by a pseudo-static analysis or SIPD calculation is a way to deal with inertial instability.

By an extensive review of existing methods, Murphy (2010) defined three approaches to estimate seismic induced displacements: rigid-block, decoupled and coupled analysis. For the rigid-block analysis, the Newmark (1965) method is the most recognizable; the Makdisi and Seed (1978) is one

of the most used decoupled methods; and the Bray and Travasarou (2007) method as well as numerical dynamic analyses performed by software such as PLAXIS or FLAC are part of the coupled methods.

In order to tackle the methods mentioned above, a review of the seismic response of the materials involved with the analysis should be accounted, mainly for coarse tailings. New information regarding dynamic behaviour of comparable materials is available and the current state of art of the seismic response analysis has greatly developed the last decade, as remarked by Stewart et al. (2008) and Hashash et al. (2010). The following sections describe the theoretical background of 1D seismic response analysis and SIPD calculations.

### **1D seismic response analysis**

The surface seismic response of an earthquake is greatly influenced by site soil conditions. In order to quantify such response, seismic response analyses are used to determine the dynamic soil behaviour due to the shake of the rock immediately beneath of it (Kramer, 1996). To quantify the seismic response of a rock, a seismic hazard study is performed. Dynamic behaviour of rock is less influenced by the earthquake nature due to its large stiffness. 1D seismic response analysis are based on the hypothesis that all the soil boundaries are horizontal and that soil response is particularly affected by seismic shear waves, whose propagation turns vertical as it approaches the surface.

The analysis methodology depends on how the soil behaviour is modelled. A linear analysis (LM) relies on the use of transfer functions in the frequency domain. However, the nonlinear behaviour of soils, which contrasts with the linear assumption of the LM approach, makes this methodology quite restricted. In order to account for such restriction, a simple iterative process involving dynamic equivalent linear properties of soil can be used; this methodology is called the equivalent linear approach (ELM). As mentioned, this methodology is still linear up to some extent since it focuses on searching the elastic parameters of the soil. These parameters should be consistent with seismic induced shear strain levels for each soil layer involved in the analysis.

A fully nonlinear analysis (NLM) is capable of modelling the hysteretic behaviour of soils due to earthquake loading. It uses a direct numerical integration in the time domain. Through this analysis, a linear or nonlinear stress-strain relationship can be followed by a number of small incremental linear steps. Such relationship is generally modelled by a hyperbolic model.

The load, unload and reload conditions, generally known as the extended 4 Masing (1926) rules, of the soil under cyclic loading was observed and proved by Matasovic (1993) using the DMOD (2012) software. Currently, Hashash et al. (2010) has greatly improved the deficiencies encountered when using the NLM approach (Stewart et al., 2008) by the development the Deepsoil software (Hashash, 2012).

### **Seismic induced permanent displacements**

#### ***Newmark (1965) and Houston et al. (1987)***

Newmark (1965) was the first to formulate the rigid-block analogy, and his methodology has been widely used to calculate SIPD for most geotechnical structures. The Newmark method considers a rigid block mass sliding on an inclined plane, whose SIPD equals the double integration of the difference between earthquake acceleration and a yield acceleration ( $k_y$ ), the latter concept refers to

the overall slope dynamic resistance, which depends primarily on the dynamic strength of the material along the critical sliding surface and the structure's geometry and weight (Bray, 2007). Several authors have modified the original Newmark (1965) method to overcome simplifications such as the inclined plane and the rigidity of both the sliding mass and slip surface assumptions.

Houston et al. (1987) modified the Newmark (1965) methodology by introducing a slip layer, whose "softened" properties would prevent accelerations within the sliding mass to exceed  $k_y$ . Accelerations that surpass  $k_y$  within the sliding mass would generate movements in it and then be inconsistent with the original assumption of the rigid-block method. Typically, the seismic record below the slip layer is obtained to calculate displacements.

#### ***Makdisi and Seed (1978)***

In their landmark paper, Makdisi and Seed (1978) formulated the decoupled method which consists of two separate steps: a dynamic response analysis and a sliding response analysis. The first one is performed to quantify the accelerations experienced by the sliding mass. The second one is performed to calculate SIPD through double integration of an earthquake motion. Makdisi and Seed (1978) used average accelerations computed by the procedure of Chopra (1966) and sliding block analyses to compute SIPD of earth dams and embankments (Kramer, 1996).

Makdisi and Seed (1978) were the first to develop a series of calculation charts based on their simplified decoupled method by the analysis of three earthquake records with different magnitudes. One of their charts evaluate the seismic demand experienced by the sliding mass, as a function of the slip surface depth, main body height, and crest peak acceleration of a dam; and the other chart is employed to estimate SIPD with respect to the fundamental period of the embankment (Murphy, 2010). The Makdisi and Seed (1978) method is still widely use within the geotechnical community primarily due to its simplicity, despite the fact that it was only developed for dams and embankments.

#### ***Bray and Travasarou (2007)***

Bray and Travasarou (2007) presented a simplified coupled semi-empirical predictive model to estimate the SIPD based on the Newmark (1965) rigid-block method and numerical analysis, as a way to update the method developed by Makdisi and Seed (1978). This procedure involves a block failure model sliding over a nonlinear coupled surface (Rathje and Bray, 2000), which can represent the dynamic behavior of structures such as: dams, natural slopes, compacted fill dykes and municipal solid waste fills (MSWF).

Bray and Travasarou (2007) noted that the major uncertainty for the evaluation of an earth structure is the seismic event. To overcome this issue, they took advantage of over 688 earthquake records and concluded that the spectral acceleration at a degraded period of the potential sliding mass is the most efficient and sufficient single ground motion intensity measure. The method captures the slope seismic resistance through its  $k_y$  and initial fundamental period. Using these parameters as input, Bray and Travasarou (2007) presented formulations to estimate SIPD and to evaluate the probability of negligible SIPD. Finally, they showed that their estimates were generally consistent with 16 documented cases of earth dams and MSWF.

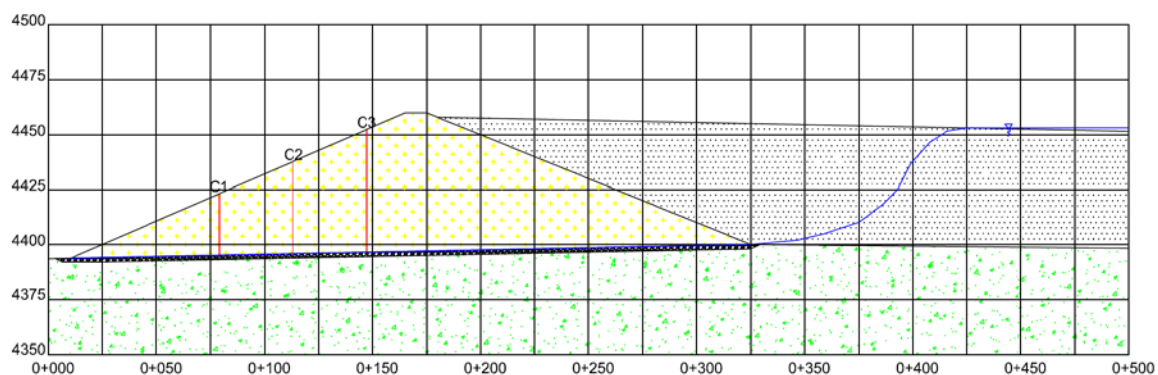
### Pseudo-static analysis

This approach consists on performing a slope stability analysis, usually by the limit equilibrium method, where a two-dimensional (2D) factor of safety (FS) is computed in which a static horizontal inertial force is applied to the potential sliding mass. This force, expressed as the product of a seismic coefficient ( $k$ ) and the potential sliding mass weight, represents the destabilizing effects of a design earthquake to be applied to the analysed structure. Hence, the validity of this approach is based on a  $k$  value representing the seismic loading.

The pseudo-static screening procedure of Hynes-Griffin and Franklin (1984) recommends, among other things, the use of half of PGA at the site, based on their assumption that 1 m of SIPD is acceptable for most earth dams. As a consequence, this approach should not be used for structures with lower values of maximum allowable SIPD. Given the need of an appropriate method to select a seismic coefficient considering the seismic performance of a structure, Bray and Travararou (2009) presented a procedure, based on the Bray and Travararou (2007) approach, that permits to select a project-specific allowable level of SIPD, and estimates the fundamental period of the sliding mass as well as a site-dependent seismic demand (expressed in terms of spectral acceleration) so that a rational seismic coefficient can be calculated.

### CASE STUDY GEOTECHNICAL OVERVIEW AND ANALYSIS

The case study presented in this paper is a 60 m-height TSF with a global downstream slope of 1V:2.5H, as shown in Figure 1. The TSF is currently in its final configuration and is located over a medium hard rock. The TSF dike is composed of coarse tailings conventionally disposed by centrifugal equipment (cyclones); fine tailings were placed upstream. The coarse tailings dike is underlined by a gravelly drain and a pumping system keeps the beach as far as 300 m away from the crest. Piezometers and seepage analysis show a good agreement between them and support the good operation of the pumping system and the drain, guaranteeing a good water management surrounding the coarse tailings dike. These conditions support that no failure mechanism are expected to be triggered by coarse tailings liquefaction during an earthquake. Therefore, the application of 1D response analysis and SIPD methodologies previously assessed can be used, providing representative results.



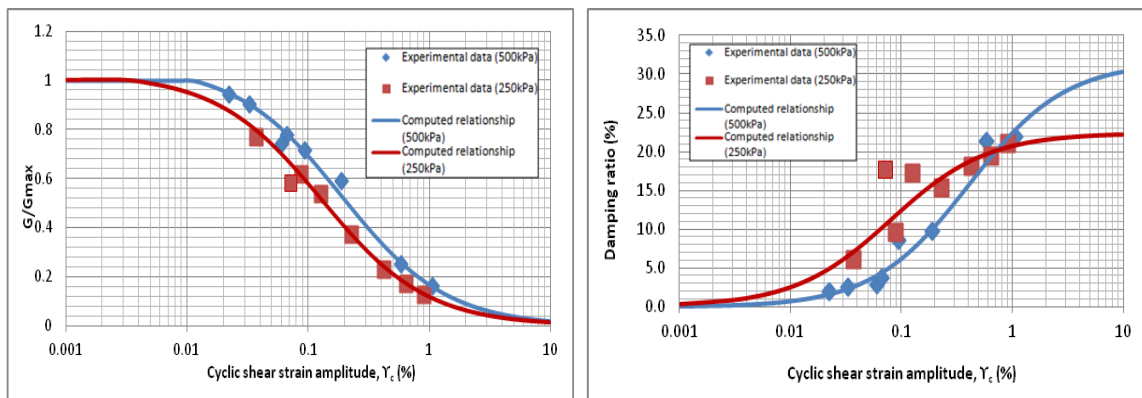
**Figure 1** Cross Section of Tailings Storage facilities

The following sections describe the geotechnical features and laboratory tests carried out on the coarse tailings. A detailed description of the geotechnical analysis performed for comparison purposes for this research is presented, which included 1D seismic response analysis and SIPD calculations using the Houston et al. (1987), Makdisi and Seed (1978) and Bray and Travarasarou (2007) methods.

### Geotechnical properties of coarse tailings

Coarse tailings dynamic properties were obtained from cyclic triaxial tests for confining pressures of 250 and 500 kPa. The shear modulus reduction and damping ratio curves were built out of 8 points, as shown in Figure 2. Finally, to extrapolate these results for shear strains from  $1 \times 10^{-3}$  to 10%, the data points were adjusted to a standard hyperbolic model.

It is important to mention that, to build a proper shear modulus reduction curve, the maximum shear modulus should be calculated through geophysical or resonant column tests since one of the limitations of cyclic triaxial testing is that the lowest shear strain to be tested is not low enough to adequately estimate the maximum shear modulus.



**Figure 2** Cyclic triaxial tests results and adjusted data

### Comparisons of dynamic curves

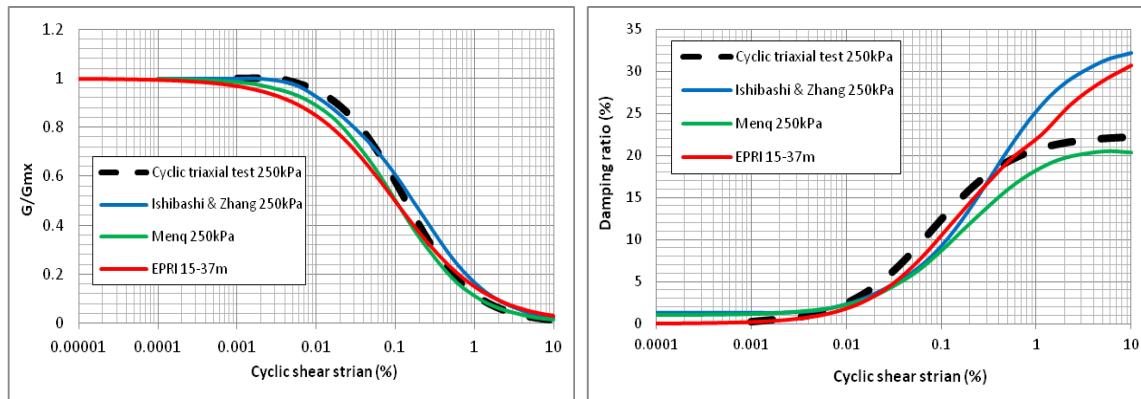
Prior to any seismic response analysis, comparisons of the dynamic curves resulting from the tests on the coarse tailings were made with existing literature curves such as Seed and Idriss (1970), Seed et al. (1986), Ishibashi Zhang (1993), Darendeli (2001) and Menq (2003). These comparisons were made to determine which ones properly model the dynamic behaviour of coarse tailings. Figure 3 shows the dynamic curves of the tested coarse tailings and the ones obtained from the Ishibashi and Zhang (1993), EPRI (1993) and Menq (2003) formulations for an average confining pressure of 250 kPa. Figure 4 shows the same curves for an average confining pressure of 500 kPa. These literature curves were selected due to its close fit with the tested material curves.

### Seismicity

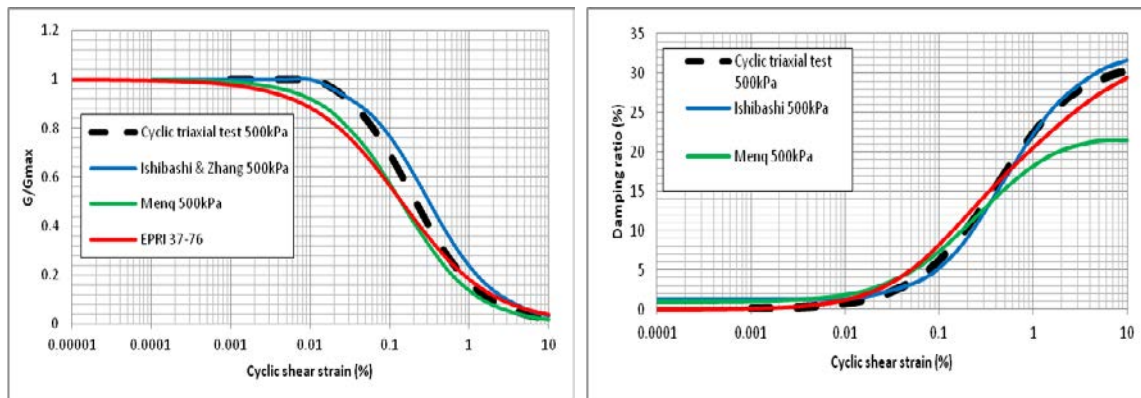
Seismic records from NS components were used as input the analyses and were obtained from published motions of subduction earthquakes recorded in Peru. The earthquake motions from the 1974 Lima, 2001 Atico, and 2007 Pisco earthquakes were chosen. No other earthquake motions were



selected due to the limited database available for Peru. The seismic records used in the seismic response were linearly scaled to PGA values of 0.18, 0.33 and 0.45 g. These PGA values were selected because they are typical from the different peruvian seismic regions. Only the seismic records scaled to a PGA value of 0.33 g were used in the determination of SIPD.



**Figure 3** Dynamic curves comparison for an average confining pressure of 250 kPa



**Figure 4** Dynamic curves comparison for an average confining pressure of 500 kPa

### 1D seismic response of coarse tailings

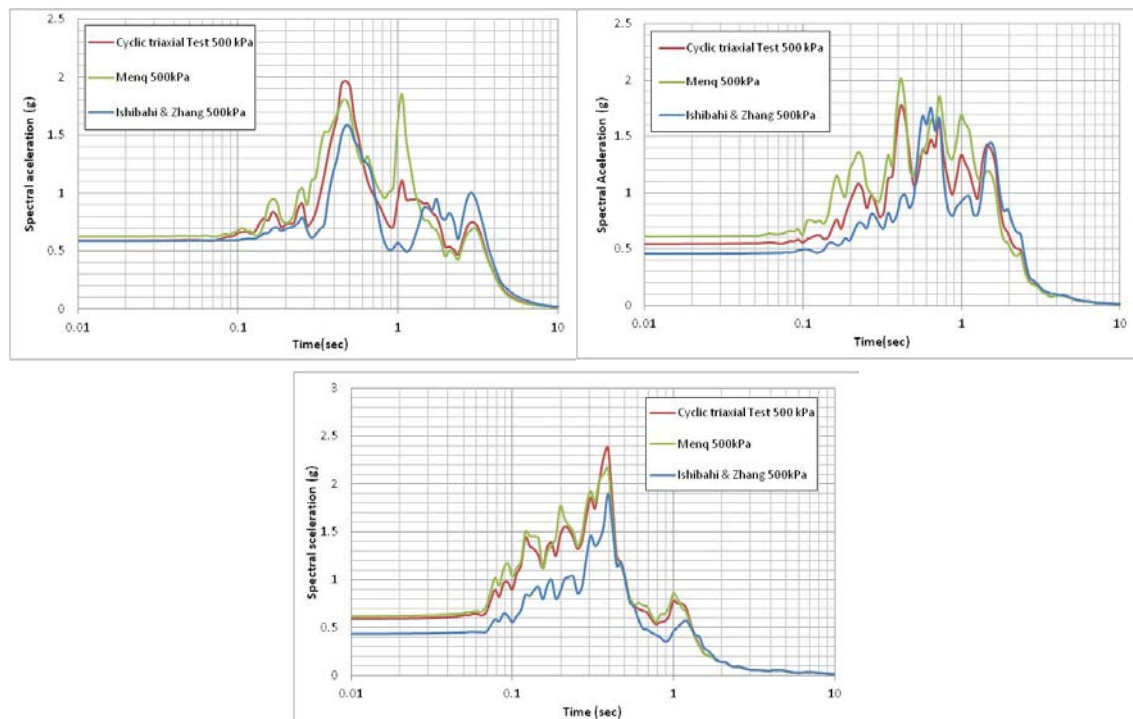
In order to define which literature based curve results in a response spectrum similar to the one obtained when using the cyclic triaxial test results, one-layer profiles of 28 and 56 m were built to run ELM seismic response analysis. This method was preferred over the NLM because the last one would have required a detailed discretization of the soil column and, in consequence, different dynamic curves for different confining pressures. However, the ELM approach provides the same results whether a discretization is performed or not. Table 1 shows the dynamic parameters used in the ELM analysis for each column.

The results showed that the curves from the Ishibashi and Zhang (1993) and Menq (2003) formulations result in the closest response spectrum to the one calculated using the cyclic triaxial test results. However, the Menq (2003) curves are preferred and subsequently used in this paper

due to its best behaviour as the confining pressure increases; the Ishibashi and Zhang (1993) curves often encounter problems for high confining pressures.

**Table 1** Dynamic properties for the ELM analysis

Column	Average confining pressure (kPa)	G <sub>max</sub> (Kpa)	V <sub>s</sub> (m/s)	Height (m)
C1	250	86 207	217	28
C3	500	107 759	242	56



**Figure 5** Response spectrum comparisons for column 3

## SEISMIC INDUCED PERMANENT DISPLACEMENTS CALCULATION

SIPD were calculated for the seismic records linearly scaled to a PGA value of 0.33 g. The procedures used were: Makdisi and Seed (1978), Houston et al. (1987) and Bray and Travararou (2007). NLM seismic response analyses were used for the Houston et al. (1987) method and executed with the software DeepSoil (Hashash, 2012). The D-MOD (Matasovic, 1993) software was used to calculate the SIPD based upon the seismic records obtained from the response analysis. The average SIPD values of all records are presented in Table 2. The Makdisi and Seed (1978) and Bray and Travararou (2007) SIPD values are also presented in Table 2 and the Figure 6 shows the cumulative displacements of the Houston et al. (1987) analysis.

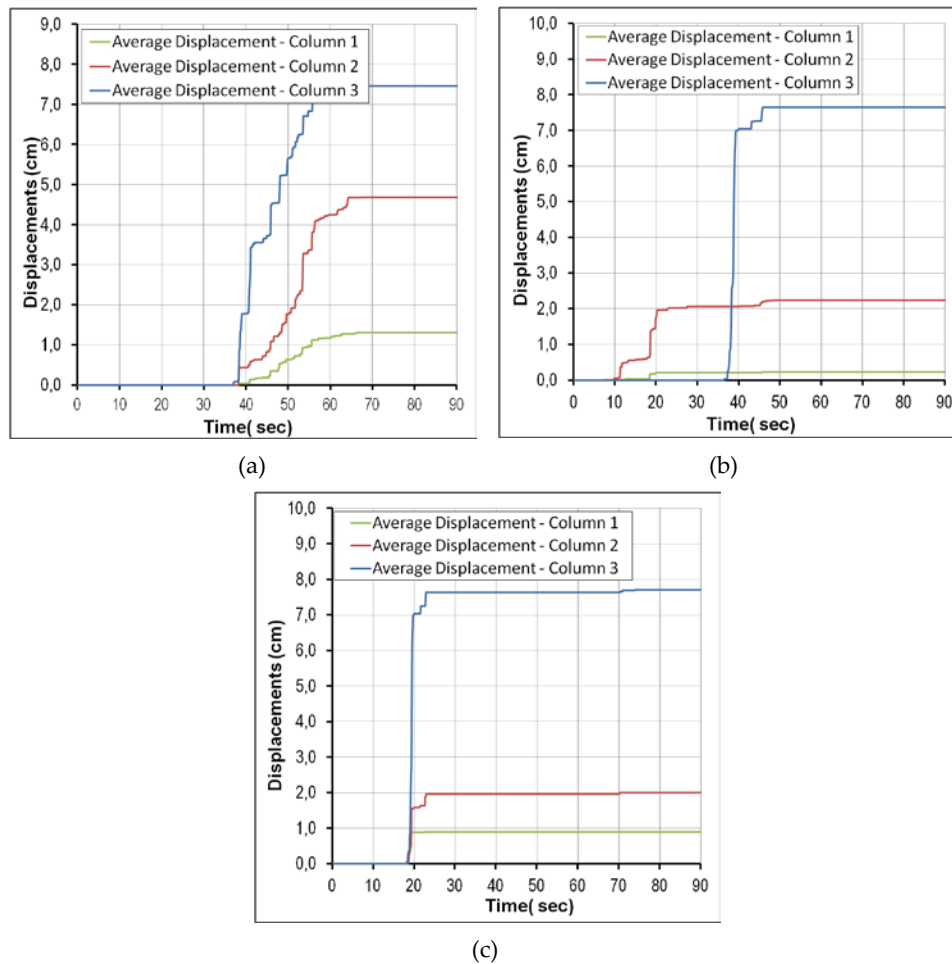
As can be seen, the Makdisi and Seed (1978) and Houston et al. (1987) methods resulted in almost negligible displacements and similar in terms of magnitude. Accordingly, the Bray and Travararou



(2007) methodology predicts a 100% probability of negligible displacements. Also, the ranges of displacements of the two first methods are within the ranges presented by Bray and Travararou (2007) except for the case the 1974 Lima earthquake where its range can be considered conservative.

**Table 2** Seismic induced permanent displacements obtained

Seismic record	Yield acceleration (g)	Seismic induced permanent displacements (cm)					
		Makdisi and Seed (1978)	Houston et al. (1987)		Bray and Travararou (2007)		
		Range	Average	Range	Probability of negligible displacements	Average	Range
1974 Lima	0.213	0.2-2.5	1.23	0.0 - 1.3	98%	10.6	5.3 - 21.2
2001 Atico	0.213	0.2-2.5	4.20	1.3 - 7.4	100%	3.8	1.9 - 7.6
2007 Pisco	0.213	0.2-2.5	3.50	0.1 - 7.7	100%	4.6	2.3 - 9.2



**Figure 6** Average seismic induced permanent displacements by Houston et al. (1987) method for the (a) 1974 Lima, (b) 2001 Atico and (c) 2007 Pisco for all columns used.

## CONCLUSIONS

A comprehensive study focusing on 1D seismic response analysis and on the estimation of SIPD of TSF composed of coarse tailings was developed. Existing literature normalized shear modulus and damping ratio curves were compared to cyclic triaxial test result on coarse tailings in order to determine which ones properly model its the dynamic behaviour. Additionally, response spectrums obtained from seismic response analysis were calculated using the same literature curves and compared to the results of the cyclic triaxial test. The results showed that the curves from the Ishibashi and Zhang (1993) and Menq (2003) formulations result in the closest fit. However, the Menq (2003) curves are preferred and subsequently used in this paper due to its best behaviour as the confining pressure increases and its relative simplicity when used in NLM analysis. Nevertheless, more testing is needed to extend this conclusion to different coarse tailings than the one tested for this research.

It was noted that, in order to obtain reliable data of the maximum shear modulus of coarse tailings, resonant column tests or geophysical tests should be carried out. Additionally, the resonant column test should be performed along with cyclic triaxial tests to adequately represent the dynamic curves of coarse tailings from small to large strains. This level of detail is particularly important when performing NLM analysis.

Several procedures for estimating SIPD on TSF were evaluated. The rigid-block Houston et al. (1987), Makdisi and Seed (1978) decoupled, Bray and Travarasrou (2007) coupled were reviewed and compared. In general, the three methodologies resulted in similar results; however, the Bray and Travarasrou (2007) was able to predict the probability of negligible displacements and resulted in rationally conservative results.

The authors recommend the use of the Bray and Travarasrou (2007) to estimate SIPD of TSF, since it involves relatively simple calculations in comparison the numerical complexity of Newmark (1965) type analysis or the use finite-element finite-difference models. However, it is important to mention that SIPD are sensitive to the fundamental period of the sliding mass and correspondent spectral acceleration, which are inputs for the Bray and Travarasrou (2007) procedure. Therefore, the determination of the dynamic characteristics of coarse tailings and a correct selection of response spectra for design are fundamental. This research suggests that the seismic design of TSF should be focused on determining SIPD rather than focused on pseudo-static factors of safety unless a rational criterion is used to define the seismic coefficient, such as the one presented by Bray and Travarasrou (2009).

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